An Aspect of Renaissance Mathematics revealed in a Study of the Theory of Human Proportion

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〔論文〕

The Theory of Human Proportion is one of the traditional theories of the fine arts. It has been produced at the interface between mathematics and the fine arts, because its theorists have considered the "Canon" (the universal "rule" of the ideal beauty of the human body) to be mathematical proportions within the human body. In order to establish the "Canon", the ultimate purpose of the theorists, they developed various types of theory of Human Proportion, using different kinds of mathematical methods. Therefore, the study of the development of the Theory of Human Proportion provides a meaningful perspective on the cultural history of mathematics.

This article discusses work on the Theory of Human Proportion in the Renaissance period. In particular, it will investigate the theory as presented in the book, *On Sculpture* written by Leon Battista Alberti. Alberti introduced a system of units (which is called the "Exempeda system"), and he attempted to establish the "Canon" of the ideal human figure through it. Through the analysis of Alberti's theory, we can clarify an aspect of Renaissance mathematics.

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I. Introduction

The Theory of Human Proportion is one of the theories of the fine arts, one which aims to establish the universal "rule" of the ideal beauty of the human body. This rule is called the "Canon", a metaphysical concept of beauty, and originates from the theory and sculptures of Polykleitos (Greek sculptor, 5c. B.C.). Contemporary art historians consider him to be the first to develop a theory of this subject. Later theorists, for example Vitruvius (1c. B.C.) in ancient Rome and Leonardo da Vinci (1452–1519) in the Renaissance, inherited the concept of the "Canon" from Polykleitos, and they attempted to produce their own new theories.

The Theory of Human Proportion has a deep relationship with mathematics, because theorists have considered the "Canon" to be a set of mathematical proportions (ratios) within the human body. The theorists used various mathematical methods to describe the ideal human body. In other words, the characteristics of the Theory of Human Proportion have been influenced by mathematics; hence the study of the development of this theory has implications not only for art history but also for the cultural history of mathematics. The analysis of this topic offers us new insights into the development of mathematics.

In this article, I will discuss the development of the Theory of Human Proportion in the Renaissance, in order to clarify one aspect of Renaissance mathematics. The works of the Renaissance theorists are significant for us, because the Renaissance produced several substantial new theories of Human Proportion, which suggest ways in which mathematics was to develop in following centuries. In particular, we will consider the theory of Leon Battista Alberti (1404–1472) who is the archetypal humanist, architect, and writer of the early Renaissance. Alberti's theories of the fine arts had a strong influence on the whole of Renaissance art.

In section 2, to distinguish some of the characteristics of the theories of the Renaissance from their predecessors, I will analyze the mathematical methods used in the theories before the Renaissance period. In section 3, I will focus on Alberti's theory as developed in the book, *On Sculpture* (*De Statua*) in the 15th century. Finally, in section 4, I will discuss some of the aspects of Renaissance mathematics on the basis of the analysis in section 3.

II. Before the Renaissance
– Three kinds of mathematical method in the Theory of Human Proportion –

As mentioned in the first section, the Theory of Human Proportion can be considered as an outgrowth of the history of the interaction between mathematics and the

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fine arts. Here, we shall discuss the mathematical elements of the Theory.

Before the Renaissance period, we find that most theories can be divided between three kinds of method. Each of these three is based on a different mathematical basis.

(1) The Fractional Method

(2) The Modulus Method

(3) The Geometrical Construction Method

These methods can be explained as follows:

(1) The Fractional Method

In this method, the length of each part of the human body is expressed as a simple integer ratio to the height of the whole body. For example, [Fig.1] shows that the length of the head is an eighth part of the height of the whole body. The "Canon" of Vitruvius is the most famous example using this method. In his book, *On Architecture* (*De architectura libri decem*), Vitruvius had developed a Theory of Human Proportion[1]. It is clear that his theory inherited the tradition of ancient Greece with its emphasis on "geometrical harmony".

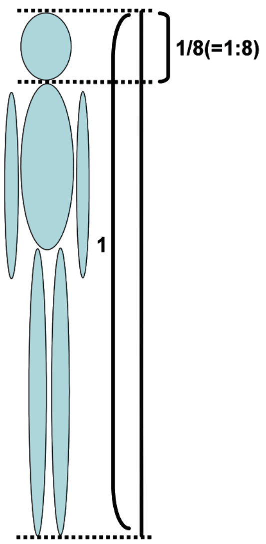
(2) The Modulus Method

In this method a standard "unit" for the height of the human body is set at the beginning. This is called the "Modulus". For example, the length of the head might be used as the Modulus. In most cases the "Modulus" is subdivided into shorter lengths in order to give the length of each of the details of the human body.

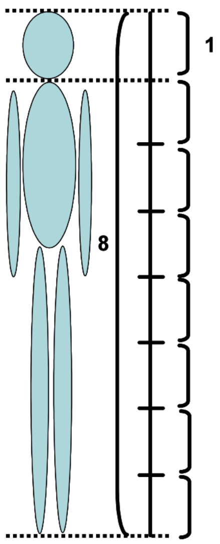
The proportion of a human figure is expressed by calculating the "Modulus". For example, in [Fig.2], the height of the whole body is fixed by using the length of the head as the "Modulus", specifically, the height is set as eight times the "Modulus". This method was been developed in the Middle Ages of Europe; Cennino d'Andrea Cennini, (Italian painter, 1370? – 1440?) had used it in his book[2], *The Craftsman's Handbook* (*Il Libro dell'Arte*). It also spread into the East, into the Byzantine and Arabic worlds. For example, in Byzantine monastery workshops, it seems to have been used until the 18^{th} century[3].

(3) The Geometrical Construction Method

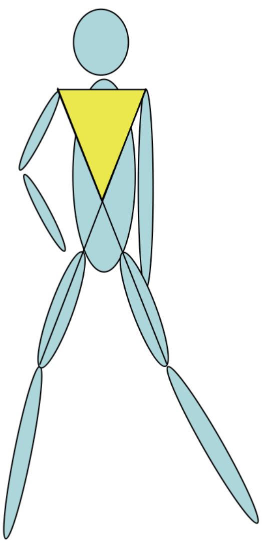
This method represents the contours and structure of the human body by using a geometrical construction (for example, in [Fig.3], the width of the shoulders and the position of the joints of the knees are fixed by constructing a triangle). With this method, plastic artists were easily able to determine the size and action of human figures. This method was particularly utilized by Gothic artisans in the Middle Ages. The "*Sketchbook*" of Villard de Honnecourt (French architect, 13c.) is a very valuable document of the methods of those times. Villard showed how to draw the outlines of creatures. His method has been called "Portraiture" [Fig.4].



[Fig.1]
The Fractional Method



[Fig.2]
The Modulus Method



[Fig.3]
The Geometrical construction Method

We have seen three kinds of mathematical methods in the Theory of Human Proportion, but it seems that the third was an epigone of the Fractional Method. Naturally, the Gothic artisans must have had a sort of geometrical knowledge, which should be called "practical geometry". However, this would probably not have been pure Greek geometry, but only a kind of practical know-how.

Therefore, at least before the Renaissance period, it seems that the mathematical methods can be divided between the Fractional Method and the Modulus Method. Let us consider these two methods further from a mathematical viewpoint. As mentioned at the beginning of this section, each of them has a different mathematical basis. The first method has geometrical thought as its basis, and it reflects one of the characteristics of Greek mathematics, a preference for continuous quantities like those of pure geometry. In contrast, the second method has an arithmetical thought as its basis, which makes us guess the influence of an inclination toward a mathematics consisting of discrete quantities, like algebra, and, as we might expect, its mathematics was particularly developed in the Eastern world.

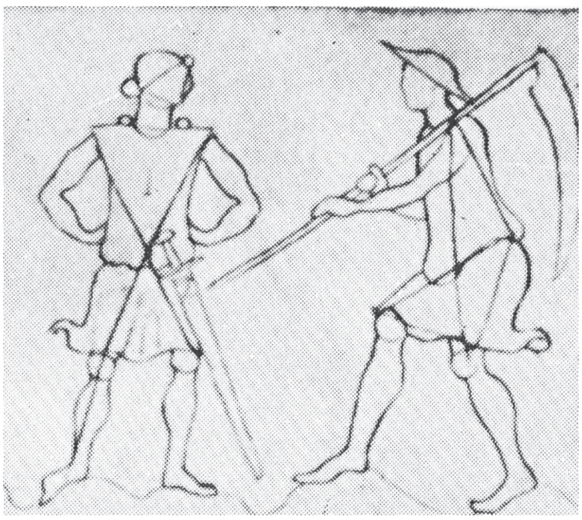
It can be seen that both mathematics and the Theory of Human Proportion have a common basis in mathematical thought.

III. Alberti's Theory
– Introducing a new mathematical method –

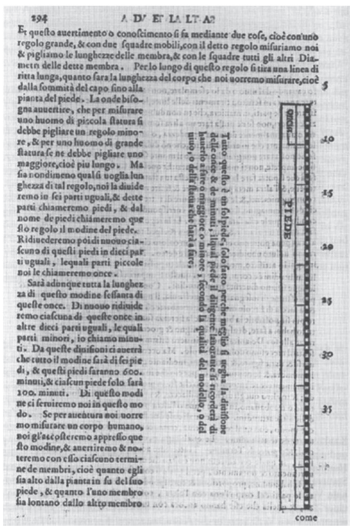
In this section, we will discuss the Theory of Human Proportion of Leon Battista Alberti. His book *On Sculpture* (*De Statua*)[4] was written in the middle of the 15th century. It can now be seen to be an epoch-making work in the Theory of Human Proportion, because Alberti introduced a completely new mathematical method, which can be called the "Quasi-Decimal Method".

In his book, Alberti suggested a precise and objective method of anthropometry for plastic artists, so that they could reproduce the human body in perfect proportions. At the beginning of the work, Alberti devised a system of units and a measuring instrument. This instrument was called the "Exempeda"; it was a sort of wooden ruler that had the same length as the height of the body ([Fig.5]). Nowadays, the system of units is called the "Exempeda system".

The "Exempeda system" has the following structure; first, an Exempeda (which is equal to the height of the body) is divided into six segments, and each segment is called a "Pedes". Secondly, a "Pedes" is divided into ten segments, each of which is called an "Unceola" (another name is the "Gradus"). Thirdly, an "Unceola" is divided into ten segments, each of which is called a "Minuta". That is to say, the "Exempeda system" consists of these four units: 1 Exempeda = 6 Pedes = 60 Unceolae = 600



[Fig.4]
Le carnet de Villard de Honnecourt (13c.), Paris, Bibliothèque nationale de France, Département des manuscrits, Français 19093, fol 37 (Études et tracés géométriques).



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Et queſto auertimento o conoſcimento ſi fa mediante due coſe, cioè con'uno regolo grande, & con due ſquadre mobili, con il detto regolo miſuriamo noi & pigliamo le lunghezze delle membra, & con le ſquadre tutti gli altri Diametri delle dette membra. Per lo lungo di queſto regolo ſi tira una linea diritta lunga, quanto ſara la lunghezza del corpo che noi uorremo miſurare, cioè dalla ſommità del capo ſino alla pianta del piede. La onde biſogna auuertire, che per miſurare uno huomo di piccola ſtatura ſi debbe pigliare un regolo minore, & per uno huomo di grande ſtatura ſe ne debbe pigliare uno maggiore, cioè piu lungo. Ma ſia nondimeno qual ſi uoglia lunghezza di tal regolo, noi la diuideremo in ſei parti uguali, & dette parti chiameremo piedi, & dal nome de piedi chiameremo queſto regolo il modine del piede. Ridiuederemo poi di nuouo ciaſcuno di queſti piedi in dieci parti uguali, lequali parti piccole noi le chiameremo once.

Sarà adunque tutta la lunghezza di queſto modine ſeſſanta di queſte once. Di nuouo ridiuideremo ciaſcuna di queſte once in altre dieci parti uguali, le quali parti minori, io chiamo minuti. Da queſte diuiſioni ci auerrà che tutto il modine ſarà di ſei piedi, & queſti piedi ſaranno 600. minuti, & ciaſcun piede ſolo ſarà 100. minuti. Di queſto modine ci ſeruiremo noi in queſto modo. Se per auentura noi uorremo miſurare un corpo humano, noi gl'accoſteremo appreſſo queſto modine, & auertiremo & noteremo con eſſo ciaſcuno termine de membri, cioè quanto egli ſia alto dalla pianta in ſu del ſuo piede, & quanto l'uno membro ſia lontano dallo altro membro

Tutto queſto è un ſol piede, ſolo fatto, perche meglio ſi uegha la diuiſione delle once & de minuti, ilqual piede il diligente lauorante ſi ricorderà di hauerlo a fare o maggiore o minore, ſecondo la qualità del modello, o del uiuo, o della ſtatura che harà a fare.

ONCIE

PIEDE

come

[Fig.5]
The "Exempeda" from L.B.Alberti, *Opuscoli morali*, 1568, p.294, Vatican, Biblioteca Apostolica Vaticana.

Minuta.

Alberti used the Exempeda system to measure many people who were selected by experts as having beautiful bodies. That is, it was possible for him to measure the length, width and thickness of each part of the human body in detail. In this manner, Alberti had gathered numerical data about "the beauty of the human body", and this data had been processed statistically in order to obtain the "Canon".

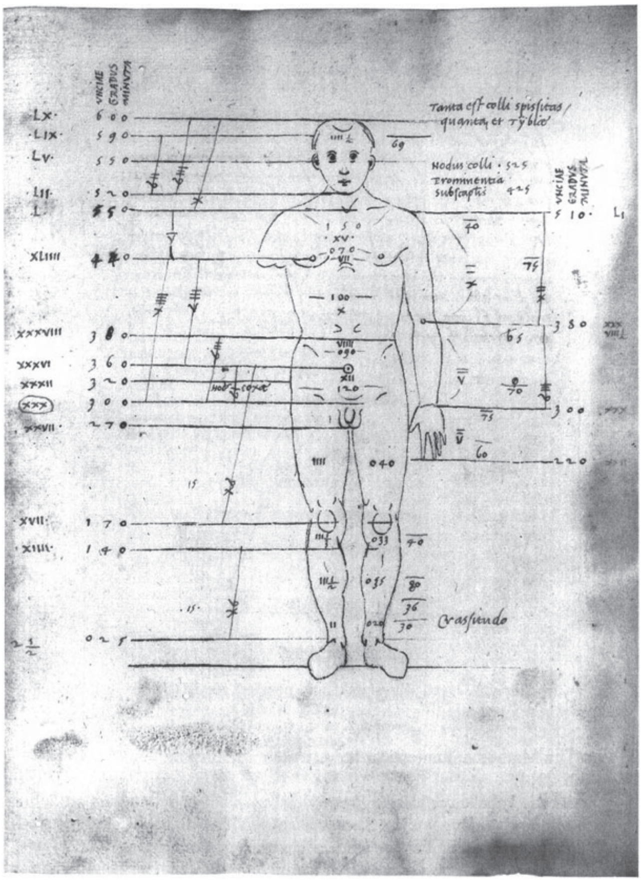
Here, we shall consider the mathematical aspects of his method. According to his book, Alberti had measured no fewer than 50 parts of the human body. The results of his measurement were made into a table. For example, [Fig.6] is an ink drawing from a manuscript of *On Sculpture* from the later 15th century, and [Table.1] is a part of the table showing the results of several measurements (of the height of each part of the body from the sole of the foot). Immediately, we can find a similar idea to that of the decimal fraction in this table. Probably, Alberti had needed a more rational and useful method for precise measurement anthropometry. This concern appears in the size of the Minuta, at 1/600 of the Exempeda. Therefore, for a person with a height of 180cm, the Minuta is only 3mm. Through fixing a small unit like this, Alberti was able to measure the details of the complex form of the human body. As an example of his goal, the far right column of the table ([Table.1]) shows the result of the measurement converted into a decimal fraction. For example, the result of the ninth row (No.9) is calculated as follows: 3 Pedes + 1 Gradus + 5 Minuta = 3/6 + 1/60 + 5/600 = 315/600 = 0.525.

This result would suggest that Alberti had required a concrete and countable quantity, so that, using it, it was possible to express as a precise quantity the size of each part of the human body.

We can thus see that the Quasi-Decimal Method was introduced in the 15th century by Alberti. This event is very important for us, because it means that the people of the Renaissance had acquired the idea of the "Number line". Of course this case is only one example; it could even be thought that the possibility of quantification had been brought about by this development. However, we cannot consider him simply as a pioneer of modern approaches, because the "Exempeda system" is a sort of relative scale, which is not based on a pure inductive method. In fact, it is not possible to ascertain whether Alberti's "Canon" was taken from an objective "scientific" method. Nevertheless, Alberti's work suggested that it might be possible for humans to use the "usefulness of mathematics" in every human activity.

IV. An aspect of the Renaissance mathematics

From the analysis in the third section, we would anticipate that the Renaissance people had used the "usefulness of mathematics" in various situations in everyday life. In that period, the Humanists and the new Renaissance "Artists" (such as Leonardo and Albrecht Dürer (1471-1528)) criticized abstract scholasticism, and they were interested in the real nature and the lived life of humans as citizens. Their effort to see nature directly must have produced a new form of knowledge. This needs to possess the ability to solve various problems connected with nature and the lived life. This must be one of the essential conditions



[Fig.6]
Alberti's Proportion figure, an ink drawing from the manuscript of *De Statua*, 15c later, Oxford, Bodleian Library, University of Oxford, Ms. Canon. Misc 172, fol. 232v.

	The Height from the sole of the foot	Pedes	Gradus	Minuta	Decimal Fraction
1	to the greatest height of the instep	0	3	0	0.05
2	to the outside of the malleolus	0	2	2	0.036···
3	to the inside of the malleolus	0	3	1	0.051···
4	to the ankle	0	8	5	0.141···
5	to the popliteal	1	4	3	0.238···
6	to the joint of knees	1	7	0	0.283 ···
7	to the testicles and the nates	2	6	9	0.448···
8	to the pubis	3	0	0	0.5
9	**to the tuber of sciatic joint**	**3**	**1**	**5**	0.525
10	to the navel	3	6	0	0.6
11	to the waist measurement	3	7	5	0.625
12	to the mammilla and the solar plexus	4	3	5	0.725
13	to the joint of the throat	5	0	0	0.833···
14	to the tuber of the neck	5	1	0	0.85
15	to the chin	5	2	0	0.866···
16	to the bottom of the thoracic vertabrae	4	2	5	0.708···
17	to the ear hole	5	5	0	0.916···
18	to the hairline above the forehead	5	9	0	0.983···
19	from the chin to the vertex	0	8	0	0.133···
20	from the chin to the ear hole	0	3	0	0.05

[Table.1]
The Table of the measurement of the human body (part)

on new knowledge, and inevitably Renaissance people required useful knowledge. Besides, a sort of practical knowledge had been brought from the Arabian world and it had been a decisive influence on the development of new knowledge in European world.

As an aspect of what we should call "humanistic knowledge", Alberti's theory had probably been formed by this process, and needless to say, it would have also played an important role in the development of mathematics after the Renaissance. From a present perspective, Alberti's method of the anthropometry is still awkward, but it has even included implicitly the concept of "Approximation".

V. Further Discussion

The people of the Renaissance developed "humanistic knowledge" by regarding real nature and phenomena as the objects of knowledge, and as they developed that, they might have noticed the power of the "usefulness of mathematics". It seems certain that the aspect discussed here is an important characteristic of Renaissance mathematics. However, we should not consider this to be its only aspect. Probably, the "completeness of mathematics" was required at the same time; that is, mathematics both possessed a usefulness and was a subject for worship. To cite an instance, this attitude of people of the Renaissance is showed symbolically in the book title of *Divine Proportion* (*Divina Proportione*), which was written by Luca Pacioli (1445-1514). Therefore, we should study further works that were produced by the interaction between mathematics and the fine arts.

Notes

1) see [ref.3], pp.159-161; "For Nature has so planned the human body that the face from the chin to the top of forehead and the roots of the hair is a tenth part; also the palm of the hand from the wrist to the top of the middle finger is as much; the head from the chin to the crown, an eighth part; from the top of the breast with the bottom of the neck to the roots of the hair, a sixth part; from the middle of the breast to the crown, a fourth part; .../ Now the navel is naturally the exact centre of the body. For if a man lies on his back with hands and feet outspread, and the centre of the circle is placed on his navel, his figure and toes will be touched by the circumference. Also a square will be found described within the figure, in the same way as a round figure is produced. For if we measure from the sole of the foot to the top of the head, and apply the measure to the outstretched hands, the breadth will be found equal to the height, just like site which are squared by "rule". (Book Ⅲ .c.i.2-3)

2) see [ref. 2], pp.48-49; "…the face is divided into these parts, namely: the forehead one: the nose, another; and from the nose to the chin, another. …From the chin under the jaw to the base of the throat, one of the three measures. The throat, one measure long. From the pit of throat to the top of the shoulder, one face; and so for the other shoulder. From the shoulder to the elbow, one face. From the elbow to the joint of the hand, one face and one the three measures….." (ChapterLXX <The Proportions which a perfectly formed man's body should possess>)

3) see [ref. 4], "Human Proportion" section2.

4) see [ref. 1], *Das Standbild* (*De Statua*), pp.142-191.

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[1] Alberti, Leon Battista (German trans., 2000), *Das Standbild* (*De Statua*) / *Die Malkunst* (*De Pictura*) / *Grundlagen der Malerei* (*Elementa Pacturae*), Herausgegeben, eingeleitet, übersetzt und kommentiert von Osker Bätshmann und Cristoph Schäublin, Darmstadt, Wissenschaftliche Buchgesellschaft.

[2] Cennini, Cennino d'Andrea (English trans., 1954), *The Craftsman's Handbook* (*Il Libro dell'Arte*) translated by Daniel V. Thompson, Jr., New York, Dover Publications Inc.

[3] Vitruvius (English trans., 1962), *On architecture* edited from the Harleian manuscript 2767 and translated by Frank Granger, Cambridge, Mass., Harvard University press.

[4] *Grove Dictionary of art* online (http://www.groveart.com), Oxford University Press.

追記

本稿は The International Conference on Mathematics Education and Cultural History of Mathematics In this Information-Oriented Society (MECHMI5, Inner Mongolia Normal University, May 19-20, 2007) における発表内容に基づく．当初は学会論文集である *Journal of the Cultural History* (*vol.9*) に掲載予定であったが，2017 年 1 月現在まで未公刊のため，雑誌編集者の了解を得て，改めて再修正の上，投稿したものである．