Measuring the Tax Revenue Elasticity to Output in a Dynamic Stochastic General Equilibrium Model

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Abstract

This paper measures the tax revenue elasticity rate and estimate a more plausible value of it using a structural method—dynamic stochastic general equilibrium (DSGE) model—with fiscal stabilization rules. In the short-run, the tax revenue elasticity to output takes a negative value, and in the medium-run, it takes considerably large range of positive values (i.e., from 2.3 to 4) in both permanent and temporary positive productivity shocks. However, in the long-run, under permanent positive productivity shocks, the tax revenue elasticity to output remains close to the current value at about 2.3, while under temporary shocks it decreases and possibly even reverses it.

Keywords: tax revenue elasticity to output, dynamic stochastic general equilibrium (DSGE) model, fiscal stabilization rules

1 Introduction

For several decades in Japan, fiscal reconstruction has been one of the major political issues. The most excited discussions revolve around the magnitude of Japanese tax revenue elasticity rate to income growth. If the income from taxes is sufficiently large, it could enable enough economic growth to reconstruct the fiscal deficit without the need for a tax rate increase, because economic growth would further increase the tax revenue. This paper investigates the magnitude of tax revenue using a structural model—a dynamic stochastic general equilibrium (DSGE) model.

Several institutions and researchers, such as OECD (2000), Cabinet Office has estimated Japanese tax revenue elasticity rate to income growth. However, some politicians and economists insist that the tax revenue elasticity is larger than these estimations and believe that fiscal reconstruction can be achieved by economic growth without a tax rate increase. Both discussions depend on theoretical ideologies, and do not have a structural economic model to support them. They deal with output growth as a secondary idea, which implies that economic growth consists of various components such as total factor productivity (TFP) growth and fiscal and monetary policy. Therefore, we propose a new method for calculating the tax revenue elasticity using the Dynamic Stochastic General Equilibrium (DSGE) model. The DSGE model is used by central banks and policy institutions to analyze economic policy, such as monetary and fiscal policy. This is a structural model, which is more comprehensive than any models used in previous research.

This paper investigates the magnitude of tax revenue elasticity to output when positive productivity shocks occur. A growth situation is considered, and how much economic growth would come from a tax revenue increase is investigated. Productivity shock is equal to total factor productivity (TFP) shock, which is the major source of cyclical and long-term economic growth. Therefore, this paper focuses on productivity shock but not on other shocks such as fiscal and monetary policies[1]. As an example of temporary positive productivity shock, the temporal deregulation is illustrated. Furthermore, as an example of a permanent shock, innovations such as the introduction of new, more productive technologies are illustrated. The study obtains three results. First, short-term impulse responses in both temporary and permanent shocks were negative. Second, medium-term impulse responses were quite large—values larger than 2.5 in both shocks. Third, the long-term impulse responses were different for temporary versus permanent shocks; the impulse responses to the temporary shock diminished, while on the other hand, the impulse responses to permanent shock were much lower. Moreover, the large value of elasticity can explain not only economic growth or the expanding tax base, but also the increasing tax rates following the rules of fiscal authority.

The rest of this paper is organized as follows: Section 2 explains the model in detail. Section 3 calibrates the model (i.e., sets the parameters and simulates the model), and Section 4 concludes.

2 The Model

This study uses the model of Iwata (2009), which is an extended variant of the medium-scale DSGE model developed by Christiano, Eichenbaum, and Evans (2005) and Smets and

Wouters (2003). The model features various real and nominal rigidities including, habit formation, investment adjustment cost, variable capital utilization, sticky prices and wages (á la Calvo (1983)), and indexation in prices and wages. Iwata (2009) includes non-Ricardian households and distortionary taxation (i.e., labor and capital income and consumption tax).

2.1 Households

There is a continuum of households indexed by $n \in [0, 1]$. A fraction $1-\omega$ of this continuum, indexed by $i \in [0, 1-\omega]$ has access to financial markets and act Ricardian; which means that the households maximize their lifetime utilities by choosing consumption and savings equal to investments in financial assets in the form of government bonds, capital stock, and utilization rate of capital stock. The remaining households, indexed by $j \in [1-\omega, 1]$, do not have access to financial markets and act non-Ricardian; which means that the households simply consume all of their current disposable income.

2.1.1 Ricardian households

Each Ricardian household i maximizes its lifetime utility by choosing consumption $C_t^R(i)$, investment $I_t(i)$, government bonds $B_t(i)$, next period's capital stock $K_t(i)$, and intensity of the capital stock utilization $z_t(i)$, given the following lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t^b \left(\frac{1}{1-\sigma_c} (C_t^R(i) - hC_{t-1}^R)^{1-\sigma_c} - \frac{\varepsilon_t^l}{1+\sigma_l} L_t^R(i)^{1+\sigma_l} \right),$$

where β is the discount factor, σ_c denotes the inverse of the intertemporal elasticity of substitution, and σ_l is the inverse of the elasticity of labor supply (i.e., labor effort) with respect to real wages. $L_t^R(i)$ represents the labor supply of the Ricardian household i. h measures the degree of habit formation in consumption. In this utility function, there are two serially correlated shocks: a preference shock ε_t^b and a labor supply shock ε_t^l. These shocks are considered and are assumed to follow a first-order autoregressive process with iid-normal error terms: $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$, $\varepsilon_t^l = \rho_l \varepsilon_{t-1}^l + \eta_t^l$.

The Ricardian household faces a real-terms flow budget constraint:

$$\begin{aligned} &(1+\tau_t^c)C_t^R(i) + I_t(i) + \Psi(z_t(i))K_{t-1}(i) + \frac{B_t(i)}{R_t P_t} \\ &= (1-\tau_t^n) w_t(i) L_t^R(i) + (1-\tau_t^k) r_t^k z_t(i) K_{t-1}(i) + (1-\tau_t^k) \frac{D_t(i)}{P_t} + \frac{B_{t-1}(i)}{P_t}, \end{aligned} \tag{1}$$

where $\Psi(z_t(i))$ represents the cost associated with variations in the degree of capital utilization $z_t(i)$. τ_t^c, τ_t^n and τ_t^k denote consumption, labor, and capital income tax rates, respectively. $D_t(i)$ denotes dividends distributed by firms to the Ricardian household i. P_t is aggregate price level, R_t is riskless return on government bonds, $w_t(i)$ is real wage, and r_t^k is real-rental rate of capital. $K_{t-1}(i)$ and $B_{t-1}(i)$ denote capital stock and government bonds of the current period in which their decisions are made at time $t-1$. For simplicity, we assume that a consumption tax is levied on private consumption expenditure alone. A lump-sum tax (or transfer) is omitted, similar to Iwata (2009).

The physical capital accumulation law of motion for the Ricardian household is expressed as follows:

$$K_t(i)=(1-\delta)K_{t-1}(i)+\left[1-S\left(\frac{\varepsilon_t^i I_t(i)}{I_{t-1}(i)}\right)\right]I_t(i), \tag{2}$$

where $\Psi(z_t(i))$ is the depreciation rate, S(・) represents the adjustment cost function in the investment, ε_t^i is a shock to investment cost function and this shock follows a first-order autoregressive (AR(1)) process as follows:

$$\varepsilon_t^i=\rho_i\varepsilon_{t-1}^i+\eta_t^i,$$

where η_t^i is an iid-normal error term. Following Iwata (2009), the capital utilization rate in a steady state is assumed as $\bar{z}=1$, and the corresponding cost as $\Psi(\bar{z})=0$. Furthermore, it is assumed that the investment adjustment cost function satisfies: $S(1)=S'(1)=0$.

Letting Λ_t and $\Lambda_t Q_t$ denote the Lagrange multipliers, the first-order conditions with respect to $C_t^R(i)$, $B_t(i)$, $I_t(i)$, $K_t(i)$, and $z_t(i)$ are expressed as follows:

$$(1+\tau_t^c)\Lambda_t=\varepsilon_t^b(C_t^R(i)-hC_{t-1}^R(i))^{-\sigma_c}, \tag{3}$$

$$\beta R_t E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\frac{P_t}{P_{t+1}}\right]=1, \tag{4}$$

$$\begin{aligned}&Q_t\left[1-S\left(\frac{\varepsilon_t^i I_t(i)}{I_{t-1}(i)}\right)\right]-Q_t S'\left(\frac{\varepsilon_t^i I_t(i)}{I_{t-1}(i)}\right)\frac{\varepsilon_t^i I_t(i)}{I_{t-1}(i)}\\&=-\beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}Q_{t+1}S'\left(\frac{\varepsilon_{t+1}^i I_{t+1}(i)}{I_t(i)}\right)\frac{\varepsilon_{t+1}^i I_{t+1}(i)^2}{I_t(i)^2}\right]+1,\end{aligned} \tag{5}$$

$$Q_t=-\beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}((1-\delta)Q_{t+1}+(1-\tau_{t+1}^k)r_{t+1}^k z_{t+1}(i)-\Psi(z_{t+1}(i)))\right]+\eta_t^q, \tag{6}$$

$$(1-\tau_t^k)r_t^k=\Psi'(z_t(i)). \tag{7}$$

Q_t represents the shadow value of an additional unit of capital, which is the same meaning of the Tobin's Q. Letting an over-bar denote a steady-state value, it can be shown that $1/\beta = \overline{R} = 1-\delta+(1-\overline{\tau}^k)\overline{r}^k+\delta\overline{\tau}^k$ and $\overline{Q} = 1$.

2.1.2 Non-Ricardian households

Non-Ricardian households are modeled as non-optimizing agents following the original assumption in Campbell and Mankiw (1989) and Galí et al (2007). Since the members of non-Ricardian household's j do not have access to financial markets, they simply consume all of their after-tax disposable income. Consumption and labor income of non-Ricardian households is denoted as $C_t^{NR}(j)$ and $L_t^{NR}(j)$, the period-by-period budget constraint they face is given by

$$(1+\tau_t^c)C_t^{NR}(j)=(1-\tau_t^n)w_t(j)L_t^{NR}(j). \tag{8}$$

2.1.3 Wage setting

Similar to Iwata (2009), it is assumed that the members of Ricardian households act as wage setters for differentiated labor services, $L_t^R(i)$, in monopolistically competitive markets[2)]. In addition, the members of non-Ricardian households are assumed to set their wages, $W_t^{NR}(j)$, for their differentiated labor services, $L_t^{NR}(j)$, to be equal to the average wage of Ricardian households. Since all households face the same labor demand schedule, the following equations are satisfied: $W_t^R(i) = W_t^{NR}(j) = W_t(n)$ and $L_t^R(i) = L_t^{NR}(j) = L_t(n)$.

An independent and perfectly competitive employment agency bundles differentiated labor, $L_t(n)$, into a single type of effective labor input, L_t, using the following technology:

$$L_t = \left[\int_0^1 L_t(n)^{\frac{1}{1+\lambda_{w,t}}} dn\right]^{1+\lambda_{w,t}},$$

where $\lambda_{w,t}$ is the wage markup, which follows a iid-normal process with drift λ_w an iid-normal error term: $\lambda_{w,t} = \lambda_w + \eta_t^w$. The employment agency solves

$$\max_{L_t(n)} W_t\left[\int_0^1 L_t(n)^{\frac{1}{1+\lambda_{w,t}}} dn\right]^{1+\lambda_{w,t}} - \int_0^1 W_t(n)L_t(n)dn,$$

where $W_t = w_t P_t$ is an aggregate nominal wage index. The labor demand schedule for each differentiated labor service is then expressed as

$$L_t(n) = \left(\frac{W_t(n)}{W_t}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t.$$

Putting the labor demand to the bundler technology of the employment agency gives the following result:

$$W_t = \left[\int_0^1 W_t(n)^{-\frac{1}{\lambda_{w,t}}} dn\right]^{-\lambda_{w,t}}.$$

With probability $1-\zeta_w$, each Ricardian household i is assumed to be allowed to optimally reset its wage. On the other hand, with probabilit ζ_w, each Ricardian household i is assumed to retain its wage. Therefore, the optimal wage $W_t^{R*}(i)$ is given by

$$W_t^{R*}(i) \equiv \underset{W_t^R(i)}{\text{argmax}} E_0 \sum_{s=0}^{\infty} (\beta\zeta_w)^s \left[\frac{1}{1-\sigma_c}(C_t^R(i) - hC_{t-1}^R(i))^{1-\sigma_c} - \frac{\varepsilon_t^l}{1+\sigma_l}\left(\left(\frac{W_t^{R*}(i)}{W_{t+s}}\right)^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} L_{t+s}\right)^{1+\sigma_l}\right],$$

subject to

$$(1+\tau_{t+s}^c)C_{t+s}^R(i) + I_{t+s}(i) + \Psi(z_{t+s}(i))K_{t+s-1}(i) + \frac{B_{t+s}(i)}{R_{t+s}P_{t+s}}$$
$$= (1-\tau_{t+s}^n)w_{t+s}(i)L_{t+s}^R(i) + (1-\tau_{t+s}^k)r_{t+s}^k z_{t+s}(i) + (1-\tau_{t+s}^k)\frac{D_{t+s}(i)}{P_{t+s}} + \frac{B_{t+s-1}(i)}{P_{t+s}}.$$

Since we know that $W_t^R(i) = W_t^{NR}(j) = W_t(n)$, aggregate nominal wage law of motion is expressed as

$$W_t = \left[(1-\zeta_w)(W_t^*(n))^{-\frac{1}{\lambda_{w,t}}} + \zeta_w\left(\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w} W_{t-1}(n)\right)\right], \tag{9}$$

where $W_t^*(n) = W_t^{R*}(i)$.

2.2 Firms

There are two types firms: perfectly competitive final-good firms and monopolistic competitive intermediate-good firms indexed by $j \in [0, 1]$. The final-good firm produces the good Y_t combining the differentiated intermediate goods, $y_t(f)$, produced by the firm f.

2.2.1 Final-good firms

The final-good producing firm combines intermediate goods using the following CES-bundler technology:

$$Y_t = \left[\int_0^1 y_t(f)^{\frac{1}{1+\lambda_{w,t}}} df\right]^{1+\lambda_{w,t}},$$

where $\lambda_{p,t}$ is the wage markup, which follows a iid-normal process with drift, and where λ_p is an iid-normal error term: $\lambda_{p,t} = \lambda_p + \eta_t^p$. The employment agency solves

$$\max_{y(f)} W_t\left[\int_0^1 y_t(f)^{\frac{1}{1+\lambda_{p,t}}} df\right]^{1+\lambda_{p,t}} - \int_0^1 p_t(f) y_t(f) df,$$

where $p_t(f)$ is the price of the intermediate good, $y_t(f)$. Then, the demand function for the intermediate good is expresses as

$$y_t(f) = \left(\frac{p_t(f)}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t.$$

Putting the demand to the CES bundler technology of the final-good firm gives a pricing rule:

$$P_t = \left[\int_0^1 p_t(f)^{-\frac{1}{\lambda_{p,t}}} df\right]^{-\lambda_{p,t}}.$$

2.2.2 Intermediate-good firms

Each intermediate-good firm f produces its differential output using an increasing return-to-scale Cobb-Douglas technology:

$$y_t(f) = \varepsilon_t^a \overline{k}_{t-1}(f)^\alpha l_t(f)^{1-\alpha} - \Phi,$$

where $\overline{k}_{t-1}(f)$ is the effective capital stock at time t, given that $\overline{k}_{t-1}(f) = z_t k_{t-1}(f)$, $l_t(f)$ is the effective labor input bundled by the employment agency and Φ represents a fixed cost. ε_t^a represents a technology shock, which follows an AR(1) process with iid-normal error term: $\varepsilon_t^a = \rho_a \varepsilon_t^a + \eta_t^a$.

Taking the real rental cost of capital, r_t^k, and aggregate real wage, w_t, cost minimization is subject to the production technology yields, marginal cost, and labor demand:

$$mc_t = \frac{w_t^{1-\alpha}(r_t^k)^\alpha}{\varepsilon_t^a \alpha^\alpha (1-\alpha)^{1-\alpha}}, \tag{10}$$

$$\frac{w_t}{r_t^k} = \frac{1-\alpha}{\alpha} \frac{z_t K_{t-1}}{L_t}. \tag{11}$$

Aggregate nominal profits are given by

$$D_t = P_t Y_t - P_t mc_t (Y_t + \Phi). \tag{12}$$

2.2.3 Price setting

Similar to the case of wage setting, sluggish price adjustment, which consists of staggered price contracts á la Calvo (1983) are assumed. A fraction, $1-\zeta_p$, of intermediate-good firms can reset (i.e., re-optimize) their prices. On the other hand, a fraction, ζ_p, does not have the opportunity of resetting their prices. Therefore, I can obtain the price indexation scheme as follows:

$$p_t(f) = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p} p_t(f),$$

where γ_p measures the degree of indexation.

An intermediate-good firm f, which is allowed to reset, knows the probability ζ_p^s that the price it chooses in this period will still be in effect s periods in the future. Taking aggregate nominal price index P_t and Y_t as given, the optimal price $p_t^*(f)$ is chosen as

$$p_t^*(f) = \underset{p_t(f)}{\operatorname{argmax}}\, E_0 \sum_{s=0}^{\infty} (\beta\zeta_p)^s \left[\left(p_t^*(f) - P_{t+s} mc_{t+s}\right) \left(\frac{P_t^*(f)}{P_{t+s}}\right)^{-\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}} Y_{t+s} - P_{t+s} mc_{t+s} \Phi \right].$$

The aggregate price law of motion is obtained as follows:

$$P_t = \left[(1-\zeta_p)\left(p_t^*(f)\right)^{-\frac{1}{\lambda_{p,t}}} + \zeta_p \left(\left(\frac{P_{t=1}}{P_{t=1}}\right)^{\gamma_p} p_{t-1}(f) \right)^{-\frac{1}{\lambda_{p,t}}} \right]^{-\lambda_{p,t}} \tag{13}$$

2.3 Fiscal and Monetary Authorities

2.3.1 Fiscal policy

The fiscal authority purchases final goods G_t, issues bonds B_t, and levies consumption, labor income, and capital income taxes at rates τ_t^c, τ_t^n and τ_t^k, respectively. The fiscal authority's real-flow budget constraint is expressed as follows:

$$G_t + \frac{B_{t-1}}{P_t} = \tau_t^c C_t + \tau_t^n w_t L_t + \tau_t^k r_t^k z_t K_{t-1} + \tau_t^k \frac{D_t}{P_t} + \frac{B_t}{R_t P_t} \tag{14}$$

This budget constraint requires fiscal rules to stabilize budgets, especially bonds. Following three feedback rules for each tax are considered and a government spending rule in log-linearlized form following Forni et al (2009) and Iwata (2009). These feedback rules are expressed as

$$\hat{\tau}_t^c = \rho_{tc} \hat{\tau}_{t-1}^c + (1-\rho_{tc})(\hat{b}_{t-1} - \hat{Y}_{t-1}) + \eta_t^{tc}, \tag{15}$$

$$\hat{\tau}_t^n = \rho_{tn} \hat{\tau}_{t-1}^n + (1-\rho_{tn})(\hat{b}_{t-1} - \hat{Y}_{t-1}) + \eta_t^{tn}, \tag{16}$$

$$\hat{\tau}_t^k = \rho_{tk} \hat{\tau}_{t-1}^k + (1+\rho_{tk})(\hat{b}_{t-1} - \hat{Y}_{t-1}) + \eta_t^{tk}, \tag{17}$$

$$\hat{G}_t = \rho_g \hat{G}_{t-1} + (1-\rho_g)(\hat{b}_{t-1} - \hat{Y}_{t-1}) + \eta_t^g, \tag{18}$$

where the hats above the variables denote log-deviations from steady states, $b_t \equiv B_t/P_t$ denotes government bond in real terms. η_t^{tc}, η_t^{tn}, η_t^{tk}, and η_t^g are iid-normal errors. It should be noted that the fiscal policy rules described here allow partial debt financing, and the debt is to be repaid through tax revenue over time. The speed of repayment is determined by a combination of the coefficients on the debt-to-output ratio of the tax and government expenditure rules, namely by the set of parameters $\rho_i,\ \phi_i (i = tc,\ tn,\ tk,\ g)$.

2.3.2 Monetary policy

The monetary authority sets nominal interest rates according to a simple feedback rule in log-linearized form:

$$\hat{R}_t = \rho_r \hat{R}_{t-1} + (1-\rho_r)\phi_{r\pi}\hat{\pi}_{t-1} + (1+\rho_r)\phi_{ry}\hat{Y}_{t-1} + \eta_t^R, \tag{19}$$

where $\pi_{t-1} \equiv \log(P_{t-1}/P_{t-2})$ denotes inflation rate. An iid-normal shock, η_t^R, to the interest rate is assumed.

2.4 Aggregation and Market Clearing

Aggregate consumption, C_t, and labor hour, L_t, in per-capita terms are given by a weighted average of the corresponding variables for each consumer type:

$$C_t = (1-\omega)C_t^R(i) + \omega C_t^{NR}(j), \tag{20}$$

$$L_t = (1-\omega)L_t^R(i) + \omega L_t^{NR}(j).$$

Since it is assumed that all households supply the same amount of labor, the aggregate labor hour is given by

$$L_t = L_t^R(i) = L_t^{NR}(j).$$

Aggregate government bonds B_t, investment I_t, physical capital K_t, and dividends D_t are expressed as

$$B_t = (1-\omega)B_t^R(i),$$

$$I_t=(1-\omega)I_t^R(i),$$

$$K_t=(1-\omega)K_t^R(i),$$

$$D_t=(1-\omega)D_t^R(i).$$

Finally, aggregate production function and the final-goods market clearing conditions are given by

$$Y_t=\varepsilon_t^a z_t K_{t-1}^{\alpha} L_t^{1-\alpha}-\Phi \tag{21}$$

$$Y_t=C_t+I_t+G_t+\Psi(z_t)K_{t-1}. \tag{22}$$

2.5 Log-Linearized Model

In this section, I log-linearize the equilibrium conditions around the steady states to induce the analytical solution. Later, over-bars denote the steady state value of the variables.

2.5.1 Ricardian households

2.5.1.1 Consumption Euler equation

From Eq. (3) and (4), we obtain the following equations:

$$\begin{aligned}\hat{C}_t^R=&\frac{h}{1+h}\hat{C}_{t-1}^R+\frac{1}{1+h}E_t\hat{C}_{t+1}^R-\frac{1-h}{(1+h)\sigma_c}(\hat{R}_t-E_t\hat{\pi}_{t+1})+\frac{1-h}{(1+h)\sigma_c}(\hat{\varepsilon}_t^b-E_t\hat{\varepsilon}_{t+1}^b)\\&-\frac{1-h}{(1+h)\sigma_c}\frac{\overline{\tau}^c}{1+\overline{\tau}^c}(\hat{\tau}_t^C-E_t\hat{\tau}_{t+}^C),\end{aligned} \tag{23}$$

where

$$\hat{\varepsilon}_t^b=\rho_b\hat{\varepsilon}_{t-1}^b+\eta_t^b. \tag{24}$$

2.5.1.2 Investment euler equation

From Eq. (5) and (4) we have

$$\hat{I}_t=\frac{1}{1+\beta}\hat{I}_{t-1}+\frac{\beta}{1+\beta}E_t\hat{I}_{t+1}+\frac{S}{1+\beta}\hat{Q}_t-\frac{\beta E_t\hat{\varepsilon}_{t+1}^i-\hat{\varepsilon}_t^i}{1+\beta}, \tag{25}$$

where $\varsigma \equiv 1/S''(1)$ and

$$\hat{\varepsilon}_t^i = \rho_i \hat{\varepsilon}_{t-1}^i + \eta_t^i. \tag{26}$$

2.5.1.3 Q equation

From Eq. (6) and (4) we have

$$\begin{aligned}\hat{Q}_t = &-(\hat{R}_t - E_t \hat{\pi}_{t+1}) + \frac{1-\delta}{1-\delta+(1-\overline{\tau}^k)\overline{r}^k} E_t \hat{Q}_{t+1} \\ &+ \frac{(1-\overline{\tau}^k)\overline{r}^k}{1-\delta+(1-\overline{\tau}^k)\overline{r}^k} E_t \hat{r}_{t+1}^k - \frac{\overline{r}^k \overline{\tau}^k}{1-\delta+(1-\overline{\tau}^k)\overline{r}^k} E_t \hat{\tau}_{t+1}^k + \hat{\eta}_t^q.\end{aligned} \tag{27}$$

2.5.1.4 Capital utilization decision equation

From Eq. (7) we have

$$\hat{z}_t = \Psi \left[\hat{r}_t^k - \frac{\overline{\tau}^k}{1-\overline{\tau}^k} \hat{\tau}_t^k \right], \tag{28}$$

where $\Psi \equiv \Psi'(1)/\Psi''(1)$.

2.5.1.5 Capital law of motion

From Eq. (2) we have

$$\hat{K}_t = (1-\delta)\hat{K}_{t-1} + \delta \hat{I}_t. \tag{29}$$

2.5.1.6 Real wage law of motion

From Eq. (9) we have

$$\begin{aligned}\hat{w}_t = &\frac{\beta}{1+\beta} E_t \hat{w}_{t+1} + \frac{1}{1+\beta} \hat{w}_{t-1} + \frac{\beta}{1+\beta} E_t \hat{\pi}_{t+1} - \frac{1+\beta\gamma_w}{1+\beta} \hat{\pi}_t \\ &+ \frac{\gamma_w}{1+\beta} \hat{\pi}_{t-1} - \frac{1}{1+\beta} \frac{(1-\beta\gamma_w)(1-\xi_w)}{\left(1+\frac{(1+\lambda_w)\sigma_l}{\lambda_w}\right)\xi_w} \\ &\times \left[\hat{w}_t - \sigma_l \hat{L}_t - \frac{\sigma_c}{1-h}(\hat{C}_t^R - hC_{t-1}^R) - \hat{\varepsilon}_t^l - \eta_t^w - \frac{\overline{\tau}^n}{1-\overline{\tau}^n} \hat{\tau}_t^n + \frac{\overline{\tau}^c}{1+\overline{\tau}^c} \hat{\tau}_t^c \right],\end{aligned} \tag{30}$$

where

$$\hat{\varepsilon}_t^l = \rho_l \hat{\varepsilon}_{t-1}^l + \eta_t^l. \tag{31}$$

2.5.2 Non-Ricardian Households

From Eq. (8) we have

$$\frac{\overline{C}^{NR}}{\overline{Y}}\left[\hat{C}_t^{NR}(1+\overline{\tau}^c)+\overline{\tau}^c\hat{\tau}_t^c\right]=\overline{w}\frac{\overline{L}}{\overline{Y}}\left[(1-\tau_t^n)(\hat{w}_t+\hat{L}_t)-\overline{\tau}^n\hat{\tau}_t^n\right]. \tag{32}$$

2.5.3 Firms

2.5.3.1 Marginal cost

From Eq. (10) we have

$$\hat{m}c_t = (1-\alpha)\hat{w}_t + \alpha\hat{r}_t^k - \hat{\varepsilon}_t^a. \tag{33}$$

2.5.3.2 Labor Demand

From Eq. (11) we have

$$\hat{L}_t = -\hat{w}_t + \hat{r}_t^k + \hat{z}_t + \hat{K}_{t-1}. \tag{34}$$

2.5.3.3 Profit Payment

From Eq. (12) we have

$$\frac{\overline{D}}{\overline{P}\,\overline{Y}}\hat{d}_t = (1-\overline{m}\,\overline{c})\hat{Y}_t - \overline{m}\,\overline{c}\,\varphi\hat{m}c_t. \tag{35}$$

2.5.3.4 Inflation Law of Motion

From Eq. (13) we have

$$\begin{aligned}\hat{\pi}_t = {} & \frac{\beta}{1+\beta\gamma_p}E_t\hat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p}\hat{\pi}_{t-1} \\ & + \frac{1}{1+\beta\gamma_p}\frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}\left[\alpha\hat{r}_t^k + (1-\alpha)\hat{w}_t - \hat{\varepsilon}_t^a + \eta_t^p\right],\end{aligned} \tag{36}$$

where

$$\hat{\varepsilon}_t^a = \rho_a \hat{\varepsilon}_{t-1}^a + \eta_t^a. \tag{37}$$

2.5.4 Fiscal and Monetary Authorities

2.5.4.1 Fiscal Policy Rules

From Eq. (14) － (18) we have

$$\frac{\overline{G}}{\overline{Y}}\hat{G}_t+\frac{\overline{B}}{\overline{P}\,\overline{Y}}(\hat{b}_{t-1}-\hat{\pi}_t)=\overline{\tau}^c\frac{\overline{C}}{\overline{Y}}(\hat{\tau}_t^c+\hat{C}_t)+\overline{\tau}^n\overline{w}\frac{\overline{L}}{\overline{Y}}(\hat{\tau}_t^n+\hat{w}_t+\hat{L}_t)$$
$$+\overline{\tau}^k\overline{r}^k\frac{\overline{K}}{\overline{Y}}(\hat{\tau}_t^n+\hat{r}_t^k+\hat{z}_t+\hat{K}_{t-1})+\overline{\tau}^k\frac{\overline{D}}{\overline{P}\,\overline{Y}}(\hat{r}_t^k+\hat{d}_t)+\beta\frac{\overline{B}}{\overline{P}\,\overline{Y}}(\hat{b}_t-\hat{R}_t), \tag{38}$$

$$\hat{\tau}_t^c=\rho_{tc}\hat{\tau}_{t-1}^c+(1-\rho_{tc})\phi_{tcb}(\hat{b}_{t-1}-\hat{Y}_{t-1})+\eta_t^{tc}, \tag{39}$$

$$\hat{\tau}_t^n=\rho_{tn}\hat{\tau}_{t-1}^n+(1-\rho_{tn})\phi_{tnb}(\hat{b}_{t-1}-\hat{Y}_{t-1})+\eta_t^{tn}, \tag{40}$$

$$\hat{\tau}_t^k=\rho_{tk}\hat{\tau}_{t-1}^k+(1-\rho_{tk})\phi_{tkb}(\hat{b}_{t-1}-\hat{Y}_{t-1})+\eta_t^{tk}, \tag{41}$$

$$\hat{G}_t=\rho_g\hat{G}_{t-1}+(1-\rho_g)\phi_{gy}(\hat{b}_{t-1}-\hat{Y}_{t-1})+\eta_t^g. \tag{42}$$

2.5.4.2 Monetary Policy Rule

From Eq. (19) we have

$$\hat{R}_t=\rho_r\hat{R}_{t-1}+(1-\rho_r)\phi_{r\pi}\hat{\pi}_{t-1}+(1-\rho_r)\phi_{ry}\hat{Y}_{t-1}+\eta_t^R. \tag{43}$$

2.5.5 Aggregation and Market Clearing

2.5.5.1 Goods Market Equilibrium Condition

From Eq. (20) we have

$$\frac{\overline{C}}{\overline{Y}}\hat{C}_t=(1-\omega)\frac{\overline{C}^R}{\overline{Y}}\hat{C}_t^R+\omega\frac{\overline{C}^{RN}}{\overline{Y}}\hat{C}_t^{NR}. \tag{44}$$

From Eq. (22) we have

$$\hat{Y}_t=\frac{\overline{C}}{\overline{Y}}\hat{C}_t+\delta\frac{\overline{K}}{\overline{Y}}\hat{I}_t+\frac{\overline{G}}{\overline{Y}}\hat{G}_t+(1-\tau_t^k)\overline{\tau}^k\frac{\overline{K}}{\overline{Y}}\hat{z}_t. \tag{45}$$

2.5.5.2 Aggregate Production Equation

$$\hat{Y}_t=\varphi(\hat{\varepsilon}_t^a+\alpha\hat{z}_t+\alpha\hat{K}_{t-1}+(1-\alpha)\hat{L}_t), \tag{46}$$

where $\varphi \equiv 1 + \Phi/\overline{Y}$.

3 Calibration

The magnitude of tax revenue elasticity to output when positive productivity shock occurs is investigated to better understand the magnitude of tax revenue elasticity. To do so, the positive productivity shocks are analyzed using the above model as well as by parameter and steady-state values estimated by previous research studies. Basically, the parameters of Iwata (2009) are quoted, through the quarterly frequency of this paper. Table 1 summarizes these parameter values.

3.1 Parameter and Steady-state Value Setting

The parameter and steady-state values of Iwata (2009), which uses the same model of this paper, is used in this study. Iwata (2009) calibrates several parameters (e.g., depreciation rate) and steady-state values (e.g., government bonds to output), and estimates structural parameter Bayesian inference using a Markov Chain Monte Carlo (MCMC) method. For this study, the calibrated and estimated posterior mean values of Iwata (2009) is chosen.

3.2 Calculating tax revenue elasticity to output

First, the tax revenue elasticity to output when positive productivity shock occurs is calculated, and is shown in the following AR (1) process:

$$\hat{\varepsilon}_t^a = \rho_a \hat{\varepsilon}_{t-1}^a + \eta_t^a .$$

The (marginal) tax elasticity to output at period t in DSGE model is defined as follows:

Consumption tax elasticity $\qquad \kappa_{c,t} = \dfrac{\hat{\tau}_t^c + \hat{C}_t}{\hat{Y}_t},$

Labor income tax elasticity $\qquad \kappa_{n,t} = \dfrac{\hat{\tau}_t^n + \hat{w}_t + \hat{L}_t}{\hat{Y}_t},$

Capital income tax elasticity $\qquad \kappa_{k,t} = \dfrac{\tau^k r^k K}{\overline{Y}} \dfrac{\hat{\tau}_t^k + \hat{r}_t^k + \hat{z}_t + \hat{K}_{t-1}}{\hat{Y}_t} + \dfrac{\tau^k d}{\overline{Y}} \dfrac{\hat{\tau}_t^k + \hat{d}_t}{\hat{Y}_t},$

Total tax elasticity $\qquad \kappa_{t,t} = \dfrac{\overline{\tau}^c \overline{C}}{\overline{Y}} \kappa_{tc} + \dfrac{\overline{\tau}^n \overline{w}\,\overline{L}}{\overline{Y}} \kappa_{tn} + \kappa_{tk} .$

The numerators of consumption, labor, and capital income tax revenue elasticity refer to

Table 1: The values of parameters

α	0.3	ρ_r	0.934
β	0.99	$\varphi_{r\pi}$	1.533
δ	0.06	φ_{ry}	0.254
λ_w	0.5	ρ_g	0.736
$\bar{\tau}^c$	0.08	φ_{gy}	0.068
$\bar{\tau}^n$	0.32	ρ_{tc}	0.507
$\bar{\tau}^k$	0.61	φ_{tcb}	0.013
h	0.465	ρ_{tn}	0.568
σ_c	1.62	φ_{tnb}	0.005
σ_l	2.113	ρ_{tk}	0.655
φ	1.904	φ_{tkb}	0.123
Ψ	0.416	ρ_a	0.518
ξ_w	0.824	$\bar{K}/\bar{Y}$	2.2
ξ_p	0.432	$\bar{B}/\bar{P}\bar{Y}$	0.6
γ_w	0.211	$\bar{C}/\bar{Y}$	0.56
γ_p	0.595	$\bar{G}/\bar{Y}$	0.16
ω	0.248		

Note: $\frac{\bar{L}}{\bar{Y}}$, $\bar{w}$, $\bar{r}^k$, and $\bar{m}\bar{c}$ are all set to be consistent with the steady state conditions implied by the model.

each of its deviation rates, while the denominator refers to the deviation rate of the output. Total tax revenue elasticity consists of elasticity that weighs their steady state values. The tax revenue elasticity is calculated using these measures in next subsection.

3.3 Calculation Results

The tax revenue elasticity to output is calculated during temporary and permanent positive productivity shocks. The impulse responses of each macroeconomics variables are shown in Appendix A.

3.3.1 Temporary productivity shock

Figure 1 shows the impulse response of tax revenue elasticity to output under temporary positive productivity shock. Figure 1 has several interesting features. First, short-run elasticity (i.e., all impulse responses in period 1) is negative. This is because temporary positive productivity shock decreases labor supply and consumption to increase future output. Second, the perk of elasticity draws close to period 4 (i.e., one year). The perk value

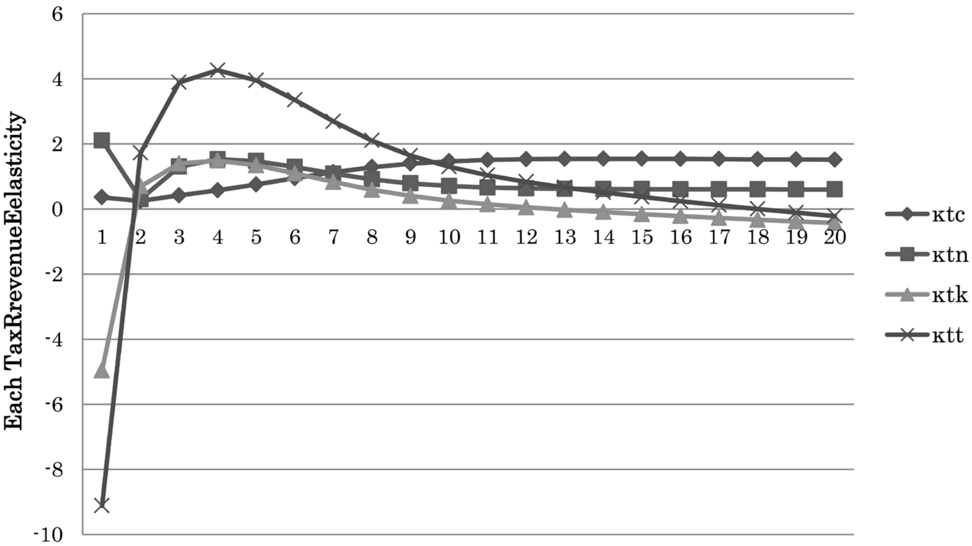

Figure 1. Impulse responses to temporary productivity shock

of total tax revenue elasticity is 4.26. Third, the values of these elasticities decrease in the long-run. This is because the tax rules work to power the decreasing tax rate via increasing output.

3.3.2 Permanent productivity shock

To analyze the permanent positive productivity shock, the transition path from the steady states of pre-shock to the state of post-shock is analyzed using the relaxation algorithm[3)]. Figure 2 shows the impulse response of tax revenue elasticity to output under permanent positive productivity shock. There are two features that can be compared with the results of temporary shock. First, although the magnitude is smaller, the short-run impulses of all elasticities are negative, similar to those of temporary shock. Second, on the other hand, the medium- and long-run responses are smoother than those of temporary shock, and the range of these values is around 2.

4 Concluding Remarks

In this paper, the tax revenue elasticity to output is shown using a DSGE model. There are two strengths to this method, which do not exist in previous research studies. First, the tax

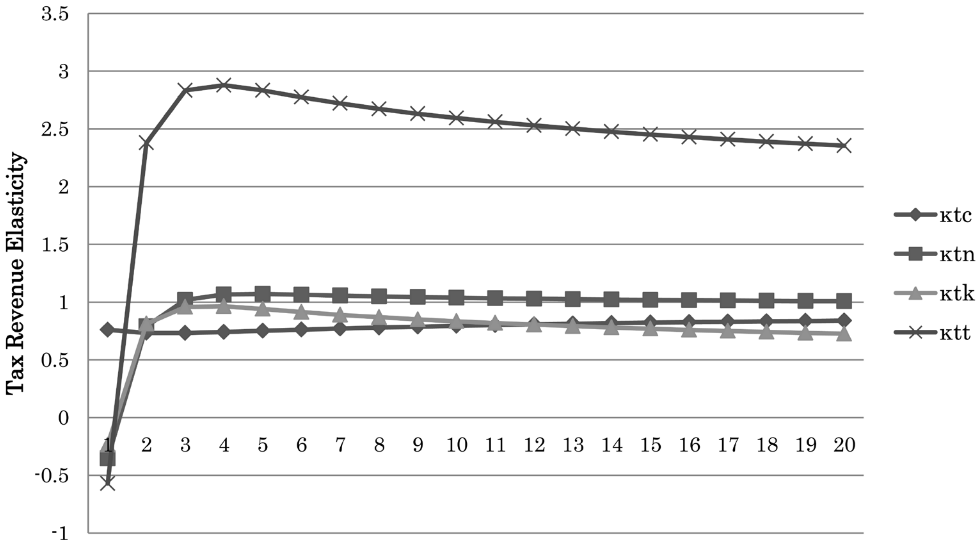

Figure 2. Impulse response to permanent productivity shock

revenue elasticity to structural shocks can be calculated, therefore allowing its contribution to economic and tax revenue growth to be identified. Second, this paper enables the investigation of the dynamic response of the tax revenue elasticity. Two policy implications were obtained. First, economic growth's response to permanent positive productivity shocks greatly increases tax revenue in the medium- and long-run, but decreases it in the short-run. Second, economic growth's response to temporary positive productivity shocks greatly increases tax revenue in the medium-run, but also decreases it in the short-and long-run. These differences come from the initial behavior of Ricardian households and tax policy rules. In the case of temporary shock, Ricardian households decrease labor supply and consumption, and therefore the total tax revenue. In the long-run, since large output growth induces a downward shift of tax rates, the total tax revenue decreases in the long-run.

There is a limitation with this finding which needs of improvement. The standard DSGE model often used, including the one in this paper, cannot analyze the fiscal reconstruction and sovereign default that many politicians and researchers are interested in, without some additions and ad-hoc assumptions[4]. I will try to investigate the tax revenue elasticity to output using a DSGE model that can analyze the sovereign default simultaneously as future research.

Footnotes:

1. Needless to say, both fiscal and monetary policy shocks affect output and tax revenue elasticity. However, both shocks are temporary and have a smaller effect to a business cycle than productivity. Moreover, these shocks may cause side effects to government debt, such as lack of fiscal discipline and inflation-financing.
2. I use the setting of Erceg et al (2006), Forni et al (2009) and Coenen and Straub (2005).
3. Trinborn, Koch and Steger (2008) detail the relaxation algorithm and provide MATLAB programs for its algorithm.
4. For example, Uribe (2006) and Bi and Traum (2012) analyze the (ad-hoc) sovereign default using the DSGE framework.

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Appendix A: Impulse responses of main macroeconomic variables to temporary and permanent positive productivity shock

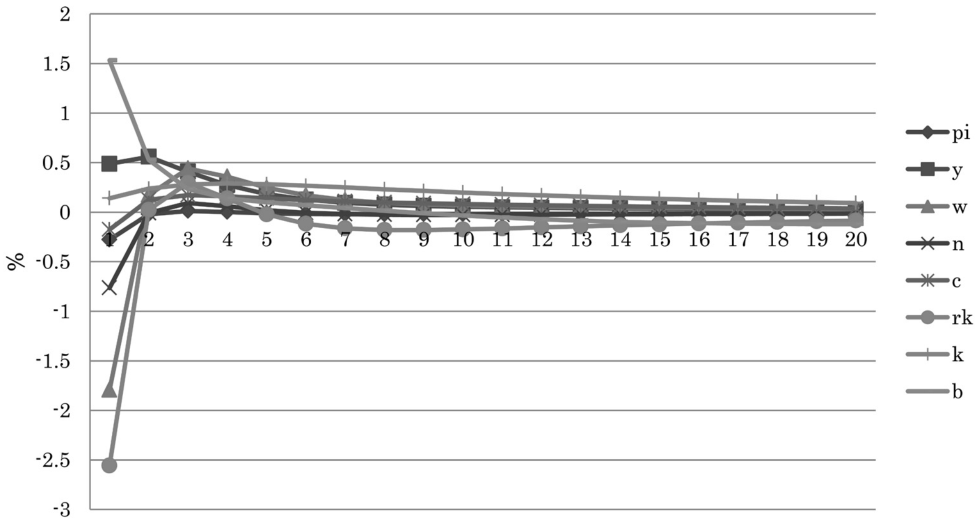

Figure A1. Impulse responses to temporary productivity shock

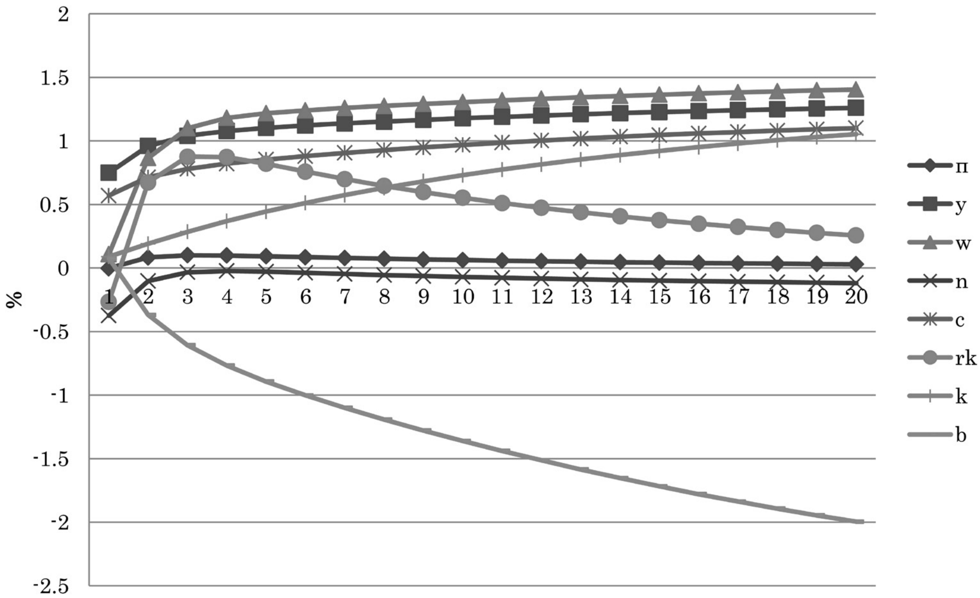

Figure A2. Impulse response to permanent productivity shock