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Effect of Fractional Flow Curves on the Recovery of Different Types of Oil in Petroleum Reservoirs

by

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Abstract

This paper considers the effect of fractional flow curves on different types of oil recovery when injecting water into petroleum reservoirs. In the computation of oil recovery, the Buckley–Leverett frontal displacement theory has been widely used to calculate the saturation profile of two immiscible fluids, wherein saturation is largely affected by the fractional flow curve of the displacing fluid. This paper reviews a fractional flow equation and a frontal advance equation and evaluates fractional flow curves of light and heavy oil by using relative permeability curves obtained from laboratory experiments. Results indicate that the fractional flow curve of light oil exhibits a regular S-shape, and application of this curve to the waterflooding method shows that a large amount of mobile oil in the reservoir is displaced by water injection. In contrast, the fractional flow curve of heavy oil does not display an S-shape because of its high viscosity. Although the advance of the injected water front is faster than that in light oil reservoirs, a significant amount of mobile oil remains behind the water front.

Keywords: Fractional flow, Relative permeability, Petroleum reservoir, Oil recovery, Waterflooding technique, Buckley–Leverett analysis

1. Introduction

In petroleum reservoir engineering, the technique of injecting water into oil reservoirs is used to maintain oil production rates during pumping operations. This so-called waterflooding technique provides high oil production rates with a high degree of petroleum recovery when applied as oil production rates begin to drop[1]. The technique has been widely employed in oil fields around the world and in shale oil exploitation technologies initiated in the United States. After long-term extraction of crude oil, a mixture of oil and water is pumped up in the production wells. Water and oil are separated by a separator installed on-site, and the separated water is recycled for injection[2].

When water is injected into a reservoir, oil is displaced toward the production well in the two-phase flow situation depicted in Fig. 1. Oil and water are mutually immiscible, so this phenomenon is referred to as immiscible displacement in porous media.

The mechanism of immiscible displacement of two-phase fluids has been studied extensively in the discipline of fluid flow through porous media. The Buckley–Leverett frontal displacement theory describes a method for calculating saturation profiles on the basis of the relative permeability, assuming that the effects of capillary pressure between the two fluids and gravity are neglected. According to this theory, the advance of a saturation front by the displacing fluid is largely affected by the permeability of oil and water relative to reservoir rock.

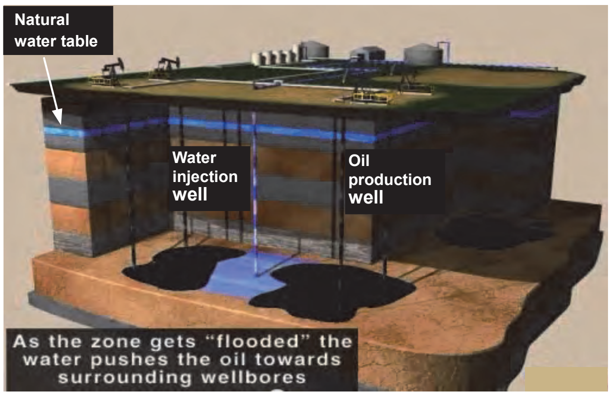

Fig. 1 Waterflooding technique in a petroleum reservoir[3].

Figure 2 illustrates typical relative permeability curves from the petroleum engineering literature[4]. The relative permeability of oil k_{ro} and water k_{rw} are generally given as a wetting fluid saturation S_w, usually that of water. As the water saturation increases, k_{ro} gradually decreases and becomes zero at residual oil saturation S_{or}. The residual oil saturation is immobile in rock

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pores. Also, k_{rw} increases as the water saturation in a medium increases, and reaches the endpoint of the relative permeability. There exists some amount S_{wi} of immobile water in pores, called the irreducible water saturation. The limits of the fractional flow are 0 % and 100 % for S_{wi} and S_{or}. At the irreducible water saturtion, the water flow rate f_w is zero and, therefore, the frational flow is 0 %. At the residual oil saturation point S_{or}, the oil flow rate is zero and the fractional flow reaches its upper limit of 100 %. The shape of the fractional flow versus the water saturation curve is characteristically an S-shape.

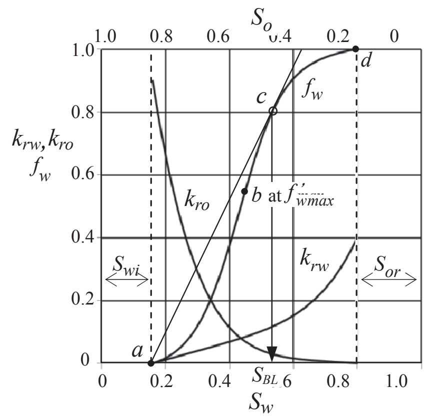

Fig. 2 Relative permeability and fractional flow rate curves[4].

There are two general methods to determine relative permeability, the steady-state (SS) method and the unsteady-state (USS) method. The SS method aims to achieve steady-state flow at different fractional flow ratios, yielding unique core saturation at each ratio. The results are easy to interpret, but it takes a long time to achieve steady-state conditions. In the USS method, the core saturated with oil is flooded by water at a constant total rate until no more oil is produced. Flooding experiments record the fractional flow ratio, the pressure at both ends, and the breakthrough time of the injected fluid. From fractional flow theory, the two-phase relative permeability can then be determined as a function of saturation at the effluent core end. For this reason, fundamental equations related to these phenomena are presented below.

2. Buckley-Leverett Analysis

2.1 *Fractional flow equation*

Derivation of the fractional flow equation for an oil–water system with one-dimensional flow is as follows: Consider displacement of oil by water in a system with dip angle α.

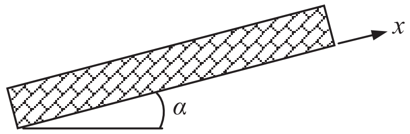

Fig. 3 One dimensional oil-water flow system.

We start with Darcy's equations

$$q_o = -\frac{kk_{ro}A}{\mu_o}\left(\frac{\partial p_o}{\partial x} + \rho_o g \sin\alpha\right), \tag{1}$$

$$q_w = -\frac{kk_{rw}A}{\mu_w}\left(\frac{\partial p_w}{\partial x} + \rho_w g \sin\alpha\right) \tag{2}$$

and replace the water pressure by $p_w = p_o - p_{cow}$, so that

$$q_w = -\frac{kk_{rw}A}{\mu_w}\left(\frac{\partial (p_o - p_{cow})}{\partial x} + \rho_w g \sin\alpha\right). \tag{3}$$

Here, p_{cow} is the capillary pressure between oil and water. After rearranging, the equations may be written as

$$-q_o \frac{\mu_o}{kk_{ro}A} = \frac{\partial p_o}{\partial x} + \rho_o g \sin\alpha, \tag{4}$$

$$-q_w \frac{\mu_w}{kk_{rw}A} = \frac{\partial p_o}{\partial x} - \frac{\partial p_{cow}}{\partial x} \rho_w g \sin\alpha. \tag{5}$$

Subtracting Eq. (4) from Eq. (5), we get

$$-\frac{1}{kA}\left(q_w \frac{\mu_w}{k_{rw}} - q_o \frac{\mu_o}{k_{ro}}\right) = -\frac{\partial p_{cow}}{\partial x} + \Delta\rho g \sin\alpha. \tag{6}$$

Substituting

$$q_T = q_w + q_o \quad , \quad f_w = \frac{q_w}{q_T} \tag{7)(8}$$

and solving for the fractional flow of water, we obtain the following expression for the fraction of flowing water:

$$f_w = \frac{1 + \dfrac{kk_{ro}A}{q_T \mu_o}\left(\dfrac{\partial p_{cow}}{\partial x} - \Delta\rho g \sin\alpha\right)}{1 + \dfrac{k_{ro}}{\mu_o}\dfrac{\mu_w}{k_{rw}}}. \tag{9}$$

For the simplest case of horizontal flow with negligible capillary pressure, the expression reduces to[5]

$$f_w = \frac{1}{1 + \dfrac{k_{ro}}{\mu_o}\dfrac{\mu_w}{k_{rw}}}. \tag{10}$$

2.2 *Buckley–Leverett equation*

For a displacement process where water displaces oil, the mass balance of water around a control volume of length Δx over a time period of Δt is considered.

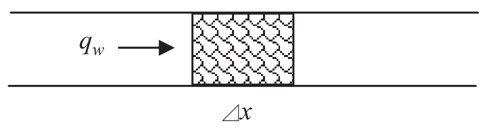

Fig. 4 Mass balance in a flow system.

The mass balance may be written as

$$\left[(q_w\rho_w)_x-(q_w\rho_w)_{x+\Delta x}\right]\Delta t=A\Delta x\phi\left[(S_w\rho_w)^{t+\Delta t}-(S_w\rho_w)^t\right] \quad (11)$$

which reduces to the continuity equation when $\Delta t\rightarrow 0$ and $\Delta x\rightarrow 0$

$$-\frac{\partial}{\partial x}(q_w\rho_w)=A\phi\frac{\partial}{\partial t}(S_w\rho_w) \quad (12)$$

Assume that the fluid compressibility may be neglected; that is, ρ_w is a constant. Also, we have that

$$f_w q_T=q_w, \quad (13)$$

so

$$-\frac{\partial f_w}{\partial x}=\frac{A\phi}{q_T}\frac{\partial S_w}{\partial t}. \quad (14)$$

Since $f_w(S_w)$, the equation may be rewritten as

$$-\frac{\partial f_w}{\partial S_w}\frac{\partial S_w}{\partial x}=\frac{A\phi}{q_T}\frac{\partial S_w}{\partial t}. \quad (15)$$

Equation (15) is known as the Buckley–Leverett equation[6].

2.3 *Frontal advance equation*

From $S_w(x,t)$, we can write the expression for saturation change as

$$dS_w=\frac{\partial S_w}{\partial x}dx+\frac{\partial S_w}{\partial t}dt. \quad (16)$$

In the Buckley–Leverett solution, we follow a fluid front of constant saturation during the displacement process as

$$0=\frac{\partial S_w}{\partial x}dx+\frac{\partial S_w}{\partial t}dt. \quad (17)$$

Substituting into the Buckley–Leverett equation, we get

$$\frac{dx}{dt}=\frac{q_T}{A\phi}\frac{df_w}{dS_w}. \quad (18)$$

Integration with respect to time as

$$\int_t \frac{dx}{dt}dt=\int_t \frac{q_T}{A\phi}\frac{df_w}{dS_w}dt \quad (19)$$

yields an expression for the position of the water front as

$$x_f=\frac{q_T t}{A\phi}\left(\frac{df_w}{dS_w}\right)_f, \quad (20)$$

which is often called the frontal advance equation[2].

3. Laboratory Experiments

Laboratory experiments for measuring relative permeability and fractional flow are performed based on the steady-state method[7,8]. Two different types of oils, light oil (kerosene) and heavy oil (type A), are used. Table 1 shows the properties of these oils and water.

Table 1 Physical property of oils and water.

Properties	Light oil	Heavy oil	Water
Density ρ (g/cm^3)	0.795	0.837	1.00
Viscosity μ (Pa·s)	0.00242	0.0167	0.001

The experimental apparatus depicted in Fig. 5 is used. In the first, the sand samples are saturated by oil, and then oil and water are simultaneously pumped at different pumping ratios. The experiment starts from high ratio of oil and low ratio of water (the imbibition process). The experiment measures the pressure and amount of discharge of oil and water.

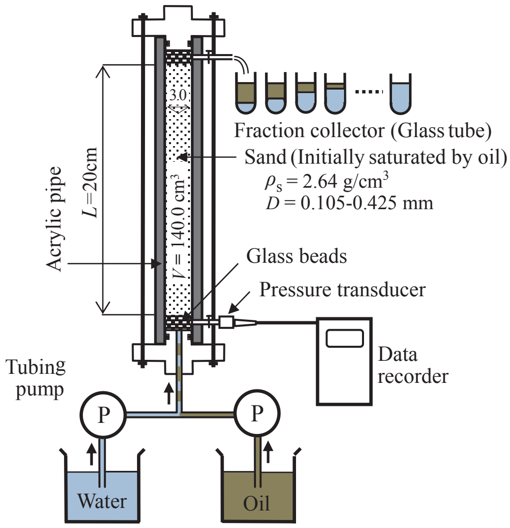

Fig. 5 Experimental apparatus for measuring relative permeabilities.

3.1 *Relative permeability and fractional flow curve of light oil*

The permeability to either fluid is expected to be lower than that for the single fluid, because it occupies only part of the pore space and may also be affected by interaction with other phases. The relative permeability of oil and water, k_{ro} and k_{rw}, are calculated by Darcy's law as

$$k_{ro}=\frac{q_o\mu_o}{kA}\Big/\frac{p}{L}\;,\quad k_{rw}=\frac{q_w\mu_w}{kA}\Big/\frac{p}{L} \quad (21)(22)$$

where p is the injection pressure, and L is the length of the sand sample. The degree of water saturation is calculated based on the fractional flow concept as

$$S_w=\int_0^{V_{ps}}(f_{od}-f_o)dV_p=1-S_o, \quad (23)$$

where f_{od} is the fractional discharge of displaced oil, and f_o is the fractional pumping ratio of oil on the total pumping rate. (For details of the calculation, see Ref. 7))

Table 2 Relative permeabilities and fractional flow data for light oil.

S_w	k_{rw}	k_{ro}	f_w
0.17	0.00	1.00	0
0.34	0.03	0.55	0.117
0.36	0.04	0.42	0.187
0.41	0.08	0.35	0.356
0.44	0.10	0.25	0.492
0.47	0.11	0.20	0.571
0.50	0.15	0.18	0.669
0.52	0.19	0.11	0.807
0.55	0.21	0.07	0.879
0.63	0.34	0.00	1.00

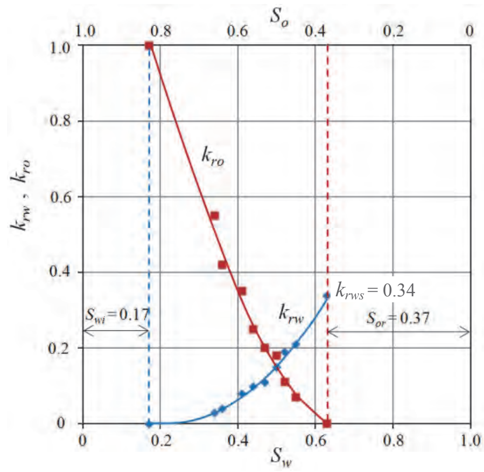

Fig. 6 Relative permeability curves for the displacement of light oil by water.

Table 2 and Fig. 6 show the change in relative permeabilities and fraction of water flow with the change in water saturation. The fractional water flow f_w was calculated from Eq. (10) using the fluid properties listed in Table 1 and the values of relative permeabilities. Relative permeability curves of light oil and water exhibit normal cross curves, and the end-point value of relative water permeability was 0.34. At that point, only water was pumped into the sand column, and a significant amount of residual oil remains in sand pores ($S_{or} = 0.37$).

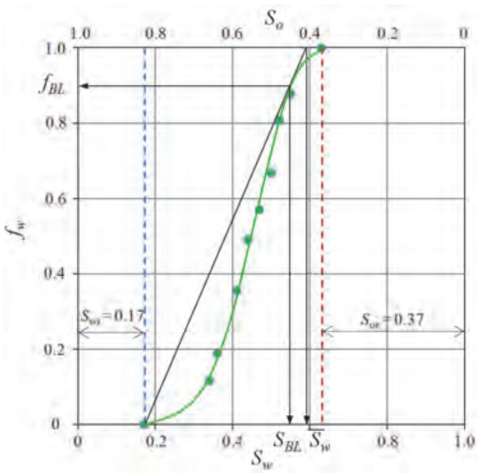

Fig. 7 Fractional flow curve in the displacement of light oil by water.

Figure 7 shows the fractional flow curve in the displacement of light oil by water. The curve has an elongated S-shape in the range of effective saturation of the displacing fluid ($S_{wi} < S_w < 1 - S_{or}$). The degree of saturation at the tangent point of a straight line drawn from the irreducible saturation on fractional flow curve S_{BL} is used to determine the saturation value at the water front according to Buckley–Leverett theory.

3.2 *Relative permeability and fractional flow curve of heavy oil*

Table 3 and Fig. 8 show the change in relative permeabilities and fractional water flow for the displacement of heavy oil by water. The data show that the residual oil saturation S_{or} is smaller than light oil, but the relative permeability of water k_{rw} is very small as compared with the light oil displacement. This is attributed to the viscosity of heavy oil being about 17 times larger than that of water, and the relative permeability of water calculated by Eq. (22) becomes very small even though water flow occurs together with oil flow through sand. Figure 9 illustrates the fractional flow curve for heavy oil. The curve does not display an S-shape; it swells on the low-saturation side, because of the very small values of k_{rw} under low water-saturation conditions.

Table 3 Relative permeabilities and fractional flow data of heavy oil.

S_w	k_{rw}	k_{ro}	f_w
0.12	0.00	0.95	0
0.15	0.01	0.77	0.178
0.18	0.01	0.64	0.207
0.24	0.02	0.54	0.382
0.28	0.03	0.45	0.527
0.34	0.03	0.36	0.582
0.44	0.04	0.27	0.712
0.50	0.04	0.19	0.779
0.60	0.05	0.10	0.893
0.73	0.06	0.00	1.00

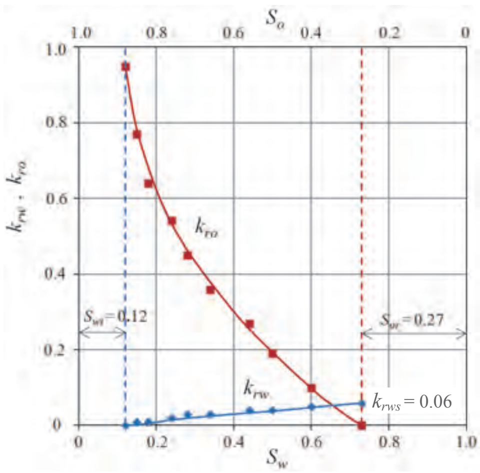

Fig. 8 Relative permeability curves for the displacement of heavy oil by water.

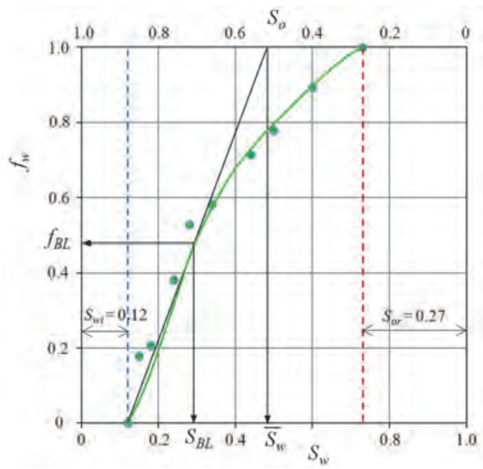

Fig. 9 Fractional flow curve in the displacement of heavy oil by water.

The difference in these fractional flow curves on the recovery of reservoir oils is investigated in the next section.

4. Application of the Fractional Flow Curves to the Waterflooding Method in Reservoirs

Advance of the saturation front in the waterflooding method may be calculated by using the frontal advance equation given by Eq. (20), which involves derivatives of the fractional rate of flow with respect to water saturation. Each saturation advances into the system at a rate in direct proportion to df_w/dS_w. The curve generally displays a smooth convex curve toward the flow direction, and the position of abrupt change in saturation, i.e., the water front, is determined from the value of S_{BL} previously shown in Figs. (7) and (9).

A petroleum reservoir of extent area A = 10 km^2, thickness B =25 m, length L = 1 km, and porosity ϕ = 0.22 is considered. The total amount of water injected is assumed to be $q_w = q_T$ = 1,000 m^3/day.

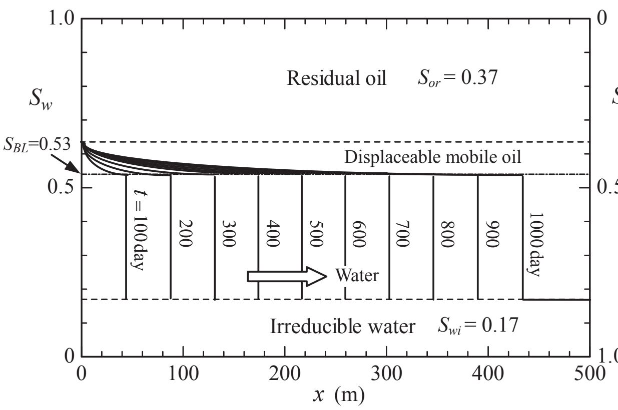

Fig. 10 Displacement of light oil calculated by the frontal advance equation.

Figure 10 illustrates the calculated results of a saturation profile for a light oil reservoir. Here, the saturation front progresses with a constant speed toward the production site (right side). Although there is a large amount of residual oil in the reservoir, water displaces the most of mobile oil. The oil recovery factor for this situation is computed as

$$RF = \frac{\bar{S}_w - S_{wi}}{1 - S_{wi}}. \tag{24}$$

From the above, RF = 0.51, from which the total amount of oil produced by waterflooding is $A\phi B \times RF = 28{,}050{,}000\,\mathrm{m}^3$ (=175 million barrels) for the given reservoir.

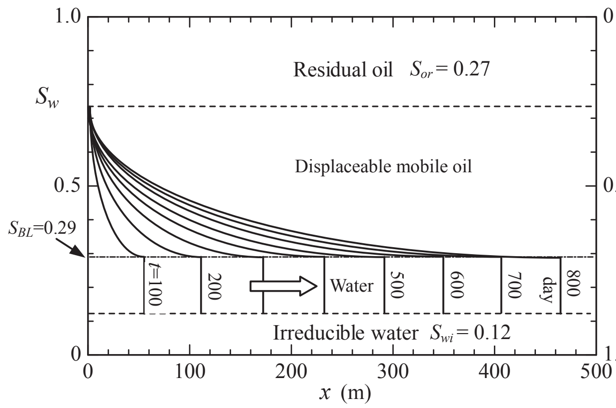

Fig. 11 Displacement of heavy oil calculated by the frontal advance equation.

Figure 11 illustrates the calculated results of the saturation profile for a heavy oil reservoir. A significant amount of mobile oil remains in the reservoir after displacement, even though the effective saturation for this displacement is larger than that for light oil. The total amount of oil produced by waterflooding is 23,100,000 m^3 (RF = 0.42) for the given reservoir.

The advance of the water front for the heavy oil reservoir is faster than that for the light oil reservoir. These results are reflected in the shape of the fractional flow curve of the reservoir oil. The viscosity ratios with water used in the waterflooding simulation are 2.42 for light oil and 16.7 for heavy oil. This suggests that the viscosity ratio between the displaced liquid and displacing liquid has a significant influence on the degree of oil recovery[9]. The use of heated water in waterflooding may be a reasonable method for improving the mobility of the reservoir oils, requiring thermodynamic analysis between the immiscible fluid flow and porous media. This might produce a challenging problem for waterflooding oil recovery.

5. Conclusion

The effect of fractional flow curves on different types of oil recovery in petroleum reservoirs was investigated in this paper. Relative permeabilities of light and heavy oils were measured through laboratory experiments, and fractional water flow was evaluated using the fractional flow equation. The fractional flow curve of light oil showed a regular S-shape, and application of this curve to the waterflooding method showed that a large amount of mobile oil in the reservoir is displaced by water injection. In contrast, the fractional flow curve of heavy oil did not display an S-shape because of its high viscosity, and a significant amount of mobile oil remains in the reservoir behind the water front. With the fractional flow data employed in this study, the oil recovery rate by waterflooding of light oil reservoirs was about 52 % and that of heavy oil reservoirs was 42 %.

Nomenclature

k	Intrinsic permeability
k_{rw}	Relative permeability of water
k_{ro}	Relative permeability of oil
k_{rws}	End-point relative permeability
μ_w	Viscosity of water
μ_o	Viscosity of oil
ρ_w	Density of water
ρ_o	Density of oil
p_w	Water pressure
p_o	Oil pressure
q_T	Total pumping rate of oil and water
q_w	Amount of water
q_o	Amount of oil
S_w	Water saturation
S_o	Oil saturation
S_{wi}	Irreducible water saturation
S_{or}	Residual oil saturation
f_w	Fractional water flow
f_o	Fractional oil flow
ϕ	Porosity of reservoir
V_p	Pore volume
A	Cross-sectional area
B	Thickness of reservoir
RF	Recovery factor

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