Proc. Schl. Eng. Tokai Univ., Ser. E
40 (2015) 77-82

Bending Magnetic Levitation Control Applying the Continuous Model of Flexible Steel Plate

by

Hikaru YONEZAWA [*1], Hiroki MARUMORI [*1], Takayoshi NARITA [*2] and Hideaki KATO [*3]

(Received on Mar. 31, 2015 and accepted on Jul. 15, 2015)

Abstract

We have proposed the levitation of an ultrathin steel plate that has been bent to an extent that has not induced plastic deformation. It has been confirmed that vibrations are suppressed and levitation performance is improved. However, when the steel plate is bent and levitated, the elastic vibration occurs. In practice, since the steel plate is very thin, elastic vibration cannot be sufficiently suppressed using only a limited number of electromagnets, and levitation performance is not always impeccable. To model a flexible thin steel plate, we apply a continuous model which expresses the motion of the plate. Applying the optimal control theory to the continuous model, the bending levitation experiments were carried out. It was concluded based upon the applied continuous model, that levitation performance become stable and desirable bending levitation performance was achieved.

Keywords: Flexible steel plate, Electromagnetic levitation system, Elastic vibration, Bending levitation control, Continuous model

1. Introduction

Thin steel plates are widely used as materials for automobiles, electric appliances, cans and other products in current industries. With various industrial demands, the surface quality of steel plates continues to be enhanced. However, because a contact conveyance using rollers is mainly adopted in the process of a thin-steel-plate production line, the problem of surface quality deterioration arises. In recent years, as a countermeasure for this problem, researches on the noncontact conveyance system with the application of electromagnetic levitation technology have become active [1-4]. In the past, our research group have constructed an electromagnetic levitation control system with which the relative distance between electromagnets and a steel plate are constantly maintained, aiming to prevent the steel plate from falling from the conveyer or coming into contact with the electromagnet during electromagnetic levitation conveyance [5]. However, as the steel plate becomes thinner, the vibration caused by minute unpredictable factors, including the nonlinearity of the attractive force of the electromagnet and the change in resistance due to heat generation by the electromagnet, makes it difficult to maintain the levitation state. Furthermore, when an ultrathin steel plate with a thickness of less than 0.3 mm is targeted for levitation, the levitation control becomes difficult because the thin plate undergoes increased flexure. To overcome these problems, we propose a levitation of an ultrathin-steel-plate that is bent to an extent that does not induce plastic deformation [6]. It has been confirmed that vibrations with mainly low frequencies are generated when a steel plate is bent and levitated. In addition, the levitation performance of steel plate is markedly improved [7]. However, when the steel plate is bent and levitated, the elastic vibration arises. In practice, since the steel plate is very thin, the elastic vibration cannot be sufficiently suppressed using only a limited number of electromagnets, and hence it is essential to eliminate the elastic vibration by applying the control theory as much as possible.

The control method used in the past studies of the authors had the advantage that the control system can be designed in a simple. However, specific vibrations that occur from being a flexible steel plate were not able to be considered by this control method. In other words, it was a model for independently feeding back the status of each of

*1 Graduate Student, Course of Mechanical Engineering
*2 Assistant Professor, Department of Electrical and Electronic Engineering, Tokyo University of Science, Suwa, Japan
*3 Assistant Professor, Department of Prime Mover Engineering

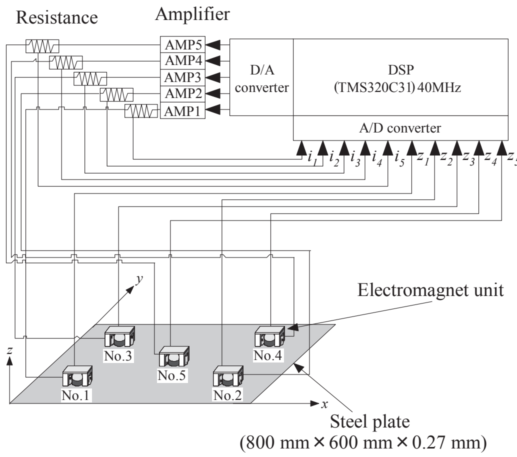


Fig. 1 Electromagnet control system

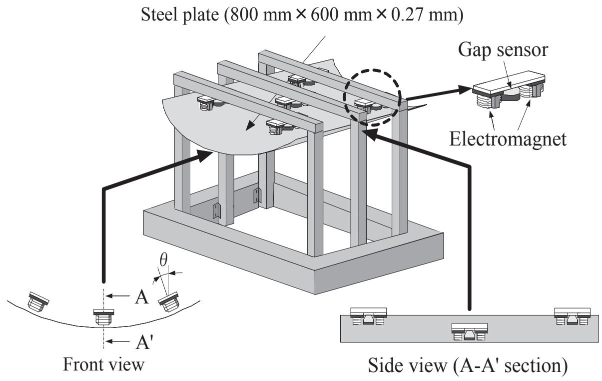


Fig. 2 Schematic illustration of experimental apparatus

the electromagnets positions. This study is intended to suppress the elastic 1st mode that is the most dominant in the vibration generated. We applied the continuous model that can be considered by integrating the state acquired at all of the electromagnets positions.

In this paper, by applying the optimal control theory to the continuous model, bending levitation experiments are carried out. We examined the levitation stability and levitation performance using a thin steel plate with a thickness of 0.27 mm.

2. Modeling of Steel Plate [5)]

In experimental apparatus, we used the same system as that used by Marumori [8)]. Figure 1 shows an outline of the electromagnet control system. Figure 2 shows a schematic illustration of experimental apparatus. In the past study, the steel plate was divided into 5 hypothetical masses and each part was modeled as a lumped constant system [6)]. The 1 degree of freedom model (1-DOF model) that has been used in past studies of the authors was not able to consider specific vibrations that occurred from being a flexible steel plate. This study is intended to suppress the elastic 1st mode that is the most dominant in the vibration generated. We applied the continuous model that can be considered by integrating the state acquired at all of the electromagnets positions.

In a continuous model, integrated control is carried out by calculating 15 values: displacement, velocity of steel plate and all the current of the electromagnet which detected at each position of the five electromagnets. In this model, the motion of the steel plate is calculated from the equations of its elastic vibration. Supporting the plate by the static attracting force of each magnet creates an equilibrium levitation state, where the steel plate maintains a certain distance from the electromagnets. The equation of small vertical motion around the equilibrium state of the steel plate subjected to magnetic forces is expressed as follows:

$$\rho h \frac{\partial^2}{\partial t^2} z + \frac{Ch^3}{12} \frac{\partial}{\partial t} \nabla^4 z + D\nabla^4 z$$
$$= \sum_{n=1}^{5} f_{cn}(t)\{\delta(x - x_{a1n})\delta(y - y_{a1n})$$
$$+\delta(x - x_{a2n})\delta(y - y_{a2n})\} \tag{1}$$

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \tag{2}$$

Where C: internal damping coefficient [Ns/m^2], $D = Eh^3/12(1\text{-}v^2)$ [Nm], v: Poisson's ratio, $f_{cn}(t)$: dynamic magnetic force at the n-th coupled magnets [N], t: time [s], $z(x, y)$: vertical displacement [m], x, y, z: coordinate axes indicated in Fig. 1 [m], x_{a1n}, x_{a2n}, y_{a1n}, y_{a2n}: location of the n-th coupled magnets (n = 1－5) [m], $\delta(\ )$: Dirac delta function [l/m].

The characteristic equations of the electromagnets can be derived as follows:

$$f_{cn} = \frac{F_n}{Z_n} z(x_{sn}, y_{sn}) + \frac{F_n}{I_n} i_n \tag{3}$$

$$\dot{i}_n = -\frac{L_{eff} I_n}{L_z Z_n^2} \dot{z}(x_{sn}, y_{sn}) - \frac{R_z}{2L_z} i_n + \frac{1}{2L_z} v_n \tag{4}$$

$$L_z = \frac{L_{eff}}{Z_n} + L_{lea} \tag{5}$$

Where F_n: magnetic force of the coupled magnets in the equilibrium state [N], I_n: current of the coupled magnets in the equilibrium state [A], x_{sn}, y_{sn}: position of the n-th sensor [m], v_n: dynamic voltage of the n-th coupled magnets [V], i_n: dynamic current of the n-th coupled magnets [A], Z_n: gap between the steel plate and electromagnet in equilibrium state [m], L_z: inductance of one magnet coil in equilibrium state [H], R_z: resistance of the coupled magnet coils [Ω], and L_{lea}: leakage inductance of the one magnet coil [H].

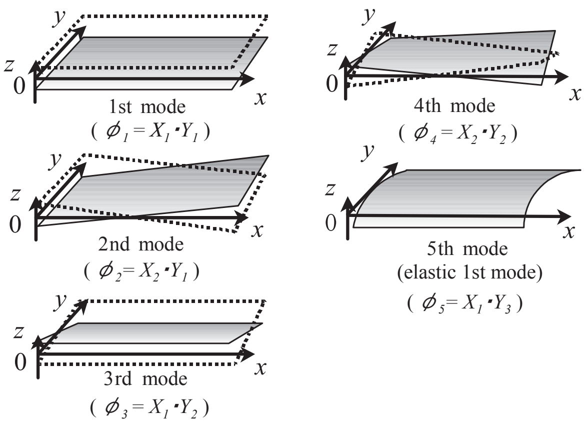


Fig. 3 Mode shapes of the levitated steel plate

3. State Equation and Controller

The vertical displacement of the plate can be expanded to an infinite series of a space-dependent eigenfunction ϕ_i (x, y) as shown in Fig. 3 multiplied by the time-dependent normal coordinate. The eigenfunctions of the plate are assumed to be products of the elastic beam eigenfunctions of the x- and y-coordinates. The function of y-coordinate $Y_{nn}(x)$ (nn = 1, 2, …) satisfies the free-free boundary condition, and the function of the x-coordinate is expressed in rigid modes (parallel and rotational motions) $X_1(y)$, $X_2(y)$ only. In addition, since the number of sensors used in this experiment are 5, we selected M = 5 for the control in which consideration is given to the 5th mode (elastic 1st mode).

$$z(x,y) = \sum_{i=1}^{M} \phi_i(x,y) W_i(t) \tag{6}$$

$$\phi_i(x,y) = X_{mm}(x) \cdot Y_{nn}(y) \quad (mm,\ nn = 1,\ 2, \cdots) \tag{7}$$

$$X_1(x) = 1 \tag{8}$$

$$X_2(x) = \frac{\sqrt{3}}{a}(2x - a) \tag{9}$$

$$Y_1(y) = 1 \tag{10}$$

$$Y_2(y) = \frac{\sqrt{3}}{b}(2y - b) \tag{11}$$

$$Y_{nn}(y) = \cos\frac{\lambda_{ynn}}{b}y + \cosh\frac{\lambda_{ynn}}{b}y + \frac{\sin\lambda_{ynn} + \sinh\lambda_{ynn}}{\cos\lambda_{ynn} - \cosh_{ynn}}\left(\sin\frac{\lambda_{ynn}}{b}y + \sinh\frac{\lambda_{ynn}}{b}y\right) \tag{12}$$

$$\cosh\lambda_{ynn} \cdot \cos\lambda_{ynn} = 1 \tag{13}$$

$$f_{ynn} = \frac{1}{2\pi}\left(\frac{\lambda_{ynn}}{b}\right)^2 \sqrt{\frac{D}{\rho h}} \tag{14}$$

State variables of the system are normal coordinates of vertical displacement of the plate $W_i(t)$ (i = 1－5), their differential values $\dot{W}_i(t)$ (i = 1－5), and dynamic currents of the coupled magnet coils $i_n(t)$ (n = 1－5). The control input of the system is the dynamic voltages of the magnets $v_n(t)$ (n = 1 － 5). Output variables of the system are vertical displacements $z_n(x_{sn}, y_{sn}, t)$ (n = 1－5). Using the state, control and output vectors, the forgoing eqs. (3)－(6) are written as following state and output equations:

$$\dot{\boldsymbol{W}} = \boldsymbol{AW} + \boldsymbol{Bv} \tag{15}$$

$$z_n = \boldsymbol{CW} \tag{16}$$

$$\boldsymbol{W} = \left[W_1 \cdots W_5\ \dot{W}_1 \cdots \dot{W}_5\ i_1 \cdots i_5\right]^{\mathrm{T}} \tag{17}$$

$$\boldsymbol{v} = \left[v_1 \cdots v_5\right]^{\mathrm{T}} \tag{18}$$

$$z_n = \left[z_1 \cdots z_5\ \ \dot{z}_1 \cdots \dot{z}_5\ \ i_1 \cdots i_5\right]^{\mathrm{T}} \tag{19}$$

Here, details of matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are omitted due to space limitations [5)].

In this study, a control system is constructed using a discrete time system; therefore, the evaluation function of a continuous system is digitized, and the optimal control law is obtained based on the optimal control theory of the discrete time system. The following discrete time system is hereby considered.

$$z_d(i+1) = \boldsymbol{\Phi} z_d(i) + \boldsymbol{\Gamma} \boldsymbol{v}_d(i) \tag{20}$$

$$\boldsymbol{\Phi} = \exp(\boldsymbol{A}T_s) \tag{21}$$

$$\boldsymbol{\Gamma} = \left\{\int_0^{T_s} \left[\exp(\boldsymbol{A}\tau)\right] d\tau\right\}\boldsymbol{B} \tag{22}$$

Here, the evaluation function of the discrete time system is expressed as follows:

$$\boldsymbol{J}_d = \sum_{i=0}^{\infty}\left[z_d(i)^{\mathrm{T}} \boldsymbol{Q}_d z_d(i) + \boldsymbol{v}_d(i)^{\mathrm{T}} \boldsymbol{r}_d \boldsymbol{v}_d(i)\right] \tag{23}$$

$$\boldsymbol{Q}_d = \begin{bmatrix} \boldsymbol{Q}_{d1} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Q}_{d2} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Q}_{d3} \end{bmatrix} \tag{24}$$

$$\boldsymbol{Q}_{d1} = \mathrm{diag}(q_1 \cdots q_5) \tag{25}$$

$$\boldsymbol{Q}_{d2} = \mathrm{diag}(q_{S1} \cdots q_{S5}) \tag{26}$$

$$\boldsymbol{Q}_{d3} = \mathrm{diag}(q_{i1} \cdots q_{i5}) \tag{27}$$

$$\boldsymbol{r}_d = r \cdot \mathrm{diag}(1\ \ 1\ \ 1\ \ 1\ \ 1) \tag{28}$$

$$\boldsymbol{M} = \boldsymbol{\Phi}^{\mathrm{T}} \boldsymbol{M\Phi} + \boldsymbol{Q}_d - \boldsymbol{\Phi}^{\mathrm{T}} \boldsymbol{M\Gamma}\left(\boldsymbol{r}_d + \boldsymbol{\Gamma}^{\mathrm{T}} \boldsymbol{M\Gamma}\right)^{-1} \boldsymbol{\Gamma}^{\mathrm{T}} \boldsymbol{M\Phi} \tag{29}$$

$$\boldsymbol{v_d} = -\boldsymbol{F}_d z_d \tag{30}$$

$$\boldsymbol{F}_d = \left(\boldsymbol{r}_d + \boldsymbol{\Gamma}^{\mathrm{T}} \boldsymbol{M\Gamma}\right)^{-1} \boldsymbol{\Gamma}^{\mathrm{T}} \boldsymbol{M\Phi} \tag{31}$$

Table 1 Symbols and values

Symbol	Value	Symbol	Value
m	9.72×10^{-1} kg	E	206 GPa
Z_n	5.00×10^{-3} m	v	0.30
R_Z	21.0 Ω	L_z	1.41×10^{-1} H
ρ	7.50×10^{3} kg/m^3	L_{lea}	9.00×10^{-2} H
d	0.43 m	L_{eff}	2.55×10^{-4} H
C	8.0×10^{7} Ns/m	x_{s1}	155 mm
x_{s2}	645 mm	x_{s3}	155 mm
x_{s4}	645 mm	x_{s5}	400 mm
y_{s1}	85 mm	y_{s2}	85 mm
y_{s3}	515 mm	y_{s4}	515 mm
y_{s5}	300 mm	T_s	1.00×10^{-3} s

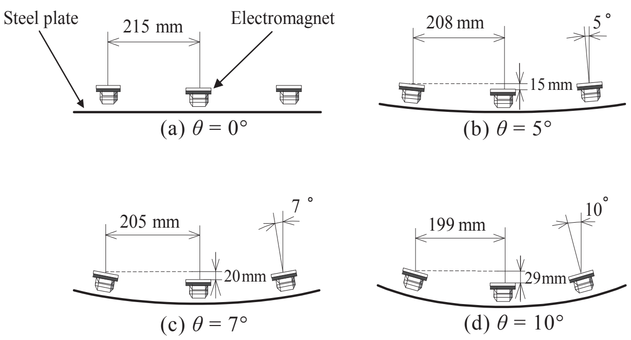


Fig. 4 Relationship between tilt angle of electromagnets and shape of steel plate

Where $\boldsymbol{Q_d}$ and $\boldsymbol{r_d}$ are weighting coefficients, $\boldsymbol{M}$ is the solution of the algebraic matrix Riccati equation, and Ts is a sampling interval. MATLAB command " lqrd " was used to solve eq. (29) and the digital controller was designed by using SIMULINK in the DSP.

4. Experiment of Bended Levitation

4.1 *Condition of experiment*

Table 1 shows the specifications of the system. Optimal control theory (OPT) is applied for levitation control of the thin steel plate to compare the results under different electromagnet tilt angles. Figure 4 shows the relationship between the tilt angle of electromagnets and shape of steel plate. In the bending levitation experiment, the electromagnet tilt angle θ is increased at intervals of 5° from 0°.

In this study, the standard deviation of displacement is measured. The standard deviation of displacement is measured 10 times at the electromagnet unit No. 1 for each electromagnet tilt angle, and the mean is used as the experimental result. It is confirmed that the same tendency is observed in other electromagnetic units. To avoid the effect of the transient state of the thin steel plate, the measurement is started approximately 10 s after the start of levitation. The weighting coefficients of OPT (eq. (23)) are set as follows:

$$q_1 - q_5 = 1.0\times10^{2}\ \mathrm{m}^{-2} \tag{32}$$

$$q_{s1} - q_{s5} = 1.0\times10^{3}\ (\mathrm{m/s})^{-2} \tag{33}$$

$$q_{i1} - q_{i5} = 10\ \mathrm{A}^{-2} \tag{34}$$

$$r = 3.83\times10^{-2}\ \mathrm{V}^{-2} \tag{35}$$

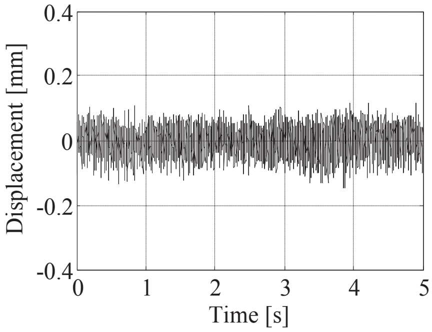
From eq. (14), natural frequency (calculated value) of elastic 1st mode = 4.23 Hz.

4.2 *Levitation experiment*

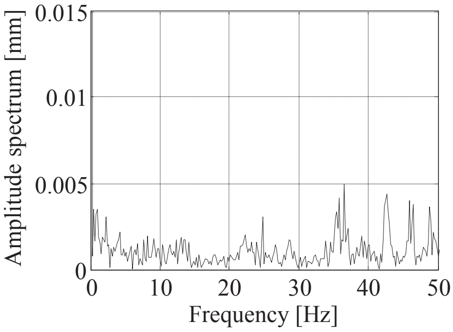
Figures 5 and 6 show the experimental results obtained when tilt angles of the electromagnets θ = 0° and 15° under continuous model. In these figures, (a) shows the displacement of the steel plate over time, (b) shows its amplitude spectrum. Figures 7 and 8 show the experimental results under 1-DOF model for comparison. In 1-DOF model levitation experiments, we used the same method as that used by Narita [6]. When 1-DOF model is applied (Figures 7 and 8), elastic vibrations are ostensible on the steel plate. This is because the elastic vibration is caused by the elastic force that is applied to the steel plate as a restoring force. A peak of the amplitude spectrum is observed at 4.23 Hz, which is the frequency of the elastic 1st mode of the steel plate used on this experiment. This is due to be a model that does not consider the vibrations excited in the steel plate. Therefore, it is confirmed that the vibration occurs in various frequencies in Fig. 7. However, the case of bent levitation steel plate in 1-DOF model (Fig. 8), only the vibration of the elastic 1st mode is mainly generated. By bending the steel plate at θ = 7°, the vibration of steel plate is suppressed compared to θ = 0°. Also, applied to continuous model, vibration suppression ability appears against the elastic 1st mode (4.23 Hz).

Figure 9 shows the relationship between the tilt angle of electromagnets θ and standard deviation of displacement. Standard deviation of displacement decreases with increasing electromagnet tilt angles. At a tilt angle of 7°, the standard deviation of displacement is the smallest. The reason behind the standard deviation of displacement of an increase at a tilt angle of 10° is that a tilt angle of 10° exceeds the natural deflection angle (8.5°) of the steel plate with a thickness of 0.27 mm, leading to difficulty in levitation.

As a result, by applying the continuous model considering the elastic vibration, it is possible to suppress the elastic 1st mode of the steel plate. Moreover, by bending the steel plate

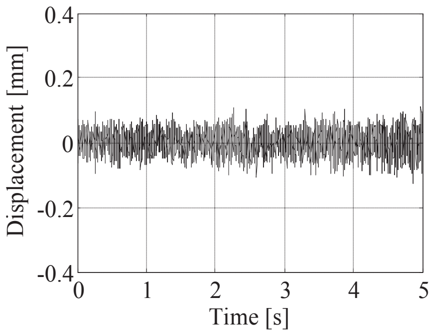


(a) Time history of displacement

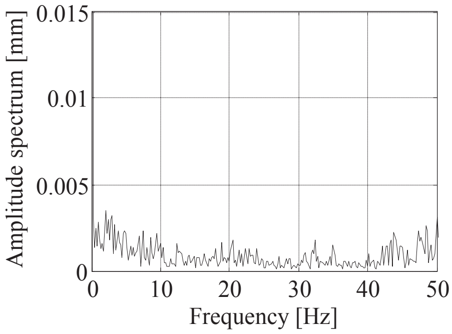


(b) Amplitude spectrum of displacement

Fig. 5 Vibration of a levitation steel plate for continuous model (θ = 0°)



(a) Time history of displacement



(b) Amplitude spectrum of displacement

Fig. 6 Vibration of a levitation steel plate for continuous model (θ = 7°)

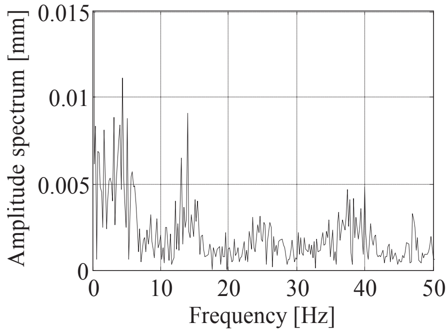


Fig. 7 Vibration of a levitation steel plate for 1-DOF model (θ = 0°)

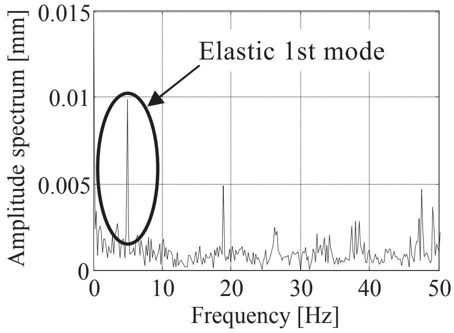


Fig. 8 Vibration of a levitation steel plate for 1-DOF model (θ = 7°)

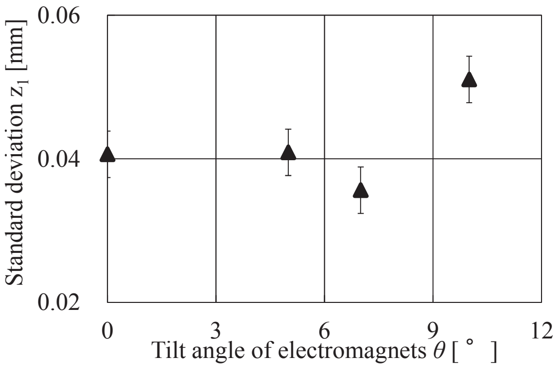


Fig. 9 Relationships between tilt angle of electromagnets θ and standard deviation of displacement at sensor No. 1

at the optimal tilt angle of the steel plate, vibration suppression performance was superior to 1-DOF model. From the above, the advantage of applying the continuous model is shown without bending the steel plate. Moreover, usefulness of bending levitation can be confirmed from experimental result with the continuous model. The model with considering elastic vibration is effective to levitate thin steel plates which easily occur the vibration.

5. Conclusion

In this paper, by applying the optimal control theory to the continuous model, bending levitation experiments were carried out. As a result, it was effective for a thin steel plate with thickness of 0.27 mm to be bent to an extent that did not exceed natural deflection angle and we were able to suppress the vibration as compared with the case of applying to the 1-DOF model. Therefore, we confirmed the utility of continuous model which takes account of elastic vibration of steel plate.

References

1) S. Matsumoto, Y. Arai, T. Nakagawa: Noncontact Levitation and Conveyance Characteristics of a Very Thin Steel Plate Magnetically Levitated by a LIM-Driven Cart, IEEE Transactions on Magnetics, 50 No. 11, (2014).

2) T. Namerikawa, D. Mizutani, S. Kuroki: Robust H∞ DIA Control of Levitated Steel Plates, IEEJ Transactions on Industry Applications, 126, No. 10 (2006), 1319-1324.

3) Jung Soo Choi, Yoon Su Baek: Magnetically-Levitated Steel-Plate Conveyance System Using Electromagnets and a Linear Induction Motor, IEEE TRANSACTIONS ON MAGNETICS, 44, No. 11 (2008), 4171-4174.

4) H. Takamine, S. Torii, T. Yanagida, S. Iwashita, S. Todoroki: Study of Electromagnetic Suspension System Using Acceleration Signal of Electromagnet Supported with Spring, IEEJ Transactions on Industry, 133, No. 5 (2008), 536-542

5) Y. Oshinoya, T. Obata: Noncontact Vibration Control of a Magnetic Levitated Rectangular Thin Steel Plate, JSME International Journal, Series C, 45, No. 1, (2002) 60-69.

6) T. Narita, H. Marumori, S.Hasegawa, Y. Oshinoya: Basic Experimental Considerations on Bending Levitation Control by Electromagnetic Force for Flexible Steel Plate, Proseedings of the School of Engineering of Tokai University, Series E, 38 (2013), 47-52.

7) H. Yonezawa, H. Marumori, T. Narita, S. Hasegawa, Y. Oshinoya: Bending Magnetic Levitation Contril for Thin Steel Plate (Experimental Consideration UsingSliding Mode Control), International Power Electronics Conference, (2014), 3055-3060.

8) H. Marumori, H. Yonezawa, T. Narita, H. Kato, S. Hasegawa, Y. Oshinoya: Effective Plate Thickness Range in Bending Technique for Levitation Control of Flexible Steel Plate, Proseedings of the School of Engineering of Tokai University, Series E, 39 (2014), 59-66.